**2017 – 2018 Log1 Contest Round 2**

**Theta Applications**

Name: _____

Units do not have to be included.

| | **4 points each** | |
|---|---|---|
| 1 | A petri dish containing bacteria halves every 2 hours. If the dish contained 56 grams of bacteria, how many grams of bacteria will be left after 4 hours, in grams? | |
| 2 | Carl travels at 75 miles per hour, traveling from NANDville to NORville. Coming back, he travels at 50 miles per hour. These two cities are 100 miles apart. What is his average speed for the round trip, in miles per hour? | |
| 3 | Tickets for the play cost \$3 for members and \$7 for their guest. To cover expenses, they must sell \$4000 worth of tickets. If 423 member-tickets are sold, how many guest-tickets must be sold to cover expenses? | |
| 4 | Nancy makes a circular pizza with an area of $4\pi$ ft$^2$. She cuts the pizza along two diameters perpendicular to each other, forming four slices of equal area. If the pizza is perfectly round, and each slice is exactly the same, find the perimeter of one slice, in feet. | |
| 5 | An airplane flies east for 150 km before turning 90° south and flying for 120 km. Find the shortest distance of the plane from its starting point, in km. | |

| | **5 points each** | |
|---|---|---|
| 6 | In 1930 the population of Waco, Texas was 630,000. The population increased to 1,430,000 by 1970, at a linear rate. Estimate its population by the year 2090. | |
| 7 | Laura is going off to college and wants to make a budget for the first semester. She comes up with the following figures: Tuition - \$2207, Food - \$855, Books - \$565, Car - \$545, Rent - \$1243, and Miscellaneous - \$800. Since Laura is organized, she constructs a pie graph to show her budget. Determine the measure of the central angle for the sector labeled Rent. | |
| 8 | A circular object with a radius of $\frac{1}{4}$ has a uniform chance of landing on any point of an 8 by 8 checkerboard. Each square on the board has a side length of 1. If the object is certain to land on the checkerboard with the possibility of overhang, find the probability that the object lands within one of the 64 squares on the checkerboard without crossing any line. | |
| 9 | Myrtle climbs to the top of the 25000-ft Mount Brie, the highest point on planet Cheddar Cheese. Ever the wise gal, she pulls out a super ball and tosses it off the top of the mountain. The ball bounces back up to 50% of its previous height with each successive bounce. Find the total vertical distance, in feet, that Myrtle's ball traveled if it continues bouncing with this pattern indefinitely. | |
| 10 | In the Snickers galaxy, there are 8 planets. On the first planet, there are 2,410 caramel people. On the second planet, 7,230 caramel people. On the third planet, 21,690 caramel people. If this pattern continues, how many total caramel people live in the galaxy? | |

| 6 points each | | |
|---|---|---|
| 11 | On the Cheese moon, a chord between two craters measure 1500 km. The two craters subtend an angle of 90 degrees at the center of the Cheese moon. Given this information, calculate the surface area of the moon. | |
| 12 | For a math team party, Black Bolt orders a large, deep-dish, Chicago-style pizza that is 26" in diameter, 4" thick in the center and uniformly increasing to 6" of thickness at the edge! Medusa decides that she wants a large slice of pizza. Therefore, she cuts out a sector that is 60 degrees. What is the volume of the slice of pizza Medusa cut, in cubic inches? | |
| 13 | A rectangle has the vertices (0,5), (2,7), (5,0), and (7,2). If a solid were to be generated by revolving the rectangle about the y-axis, what would be its volume? | |
| 14 | If Alex pulls a log with a force of 13-lbs at an angle of 60° with the horizontal and Kevin pulls the same log with a force of 16-lbs at an angle of 120° with the horizontal, what is the magnitude of the net force, in pounds, from Alex and Kevin? | |
| 15 | A sugar cone with a height of 3 cm is filled with ice cream. Above the top surface of the cone, which has a diameter of 3 cm, the ice cream forms a hemispherical, concave downward dome also with a diameter of 3 cm. Calculate the volume of all the ice cream in and above the cone. | |

**2017 – 2018 Log1 Contest Round 2**

**Alpha Applications**

Name: _____

Units do not have to be included.

| **4 points each** | | |
|---|---|---|
| 1 | A petri dish containing bacteria halves every 2 hours. If the dish contained 56 grams of bacteria, how many grams of bacteria will be left after 4 hours, in grams? | |
| 2 | Carl travels at 75 miles per hour, traveling from NANDville to NORville. Coming back, he travels at 50 miles per hour. These two cities are 100 miles apart. What is his average speed for the round trip, in miles per hour? | |
| 3 | Tickets for the play cost \$3 for members and \$7 for their guest. To cover expenses, they must sell \$4000 worth of tickets. If 423 member-tickets are sold, how many guest-tickets must be sold to cover expenses? | |
| 4 | To get from her house to the grocery store, Jane walks north along a straight path for 4 miles. She then turns right and walks east along a straight path for $\sqrt{48}$ miles. Find the bearing (or the angle E of N of the displacement) from Jane's house to the grocery store? | |
| 5 | An airplane flies east for 150 km before turning 90° south and flying for 120 km. Find the shortest distance of the plane from its starting point, in km. | |

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| 6 | In 1930 the population of Waco, Texas was 630,000. The population increased to 1,430,000 by 1970, at a linear rate. Estimate its population by thyear 2090. | |
| 7 | Laura is going off to college and wants to make a budget for the first semester. She comes up with the following figures: Tuition - \$2207, Food - \$855, Books - \$565, Car - \$545, Rent - \$1243, and Miscellaneous - \$800. Since Laura is organized, she constructs a pie graph to show her budget. Determine the measure of the central angle for the sector labeled Rent. | |
| 8 | A circular object with a radius of $\frac{1}{4}$ has a uniform chance of landing on any point of an 8 by 8 checkerboard. Each square on the board has a side length of 1. If the object is certain to land on the checkerboard with the possibility of overhang, find the probability that the object lands within one of the 64 squares on the checkerboard without crossing any line. | |
| 9 | Two doggy sleds are at the same location at $t = 0$ seconds. They start to move away from each other at an angle of 120 degrees. One doggy sled is moving at a rate of 3 m/s and the other at 5 m/s. Find the distance between the doggy sleds at $t = 2$ seconds, raised to the third power, in meters cubed. | |
| 10 | In the Snickers galaxy, there are 8 planets. On the first planet, there are 2,410 caramel people. On the second planet, 7,230 caramel people. On the third planet, 21,690 caramel people. If this pattern continues, how many total caramel people live in the galaxy? | |

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| 11 | On the Cheese moon, a chord between two craters measure 1500 km. The two craters subtend an angle of 90 degrees at the center of the Cheese moon. Given this information, calculate the surface area of the moon. | |
| 12 | For a math team party, Black Bolt orders a large, deep-dish, Chicago-style pizza that is 26" in diameter, 4" thick in the center and uniformly increasing to 6" of thickness at the edge! Medusa decides that she wants a large slice of pizza. Therefore, she cuts out a sector that is 60 degrees. What is the volume of the slice of pizza Medusa cut, in cubic inches? | |
| 13 | A rectangle has the vertices (0,5), (2,7), (5,0), and (7,2). If a solid were to be generated by revolving the rectangle about the y-axis, what would be its volume? | |
| 14 | If Alex pulls a log with a force of 13-lbs at an angle of 60° with the horizontal and Kevin pulls the same log with a force of 16-lbs at an angle of 120° with the horizontal, what is the magnitude of the net force, in pounds, from Alex and Kevin? | |
| 15 | A sugar cone with a height of 3 cm is filled with ice cream. Above the top surface of the cone, which has a diameter of 3 cm, the ice cream forms a hemispherical, concave downward dome also with a diameter of 3 cm. Calculate the volume of all the ice cream in and above the cone. | |

**2017 – 2018 Log1 Contest Round 2**

**Mu Applications**

Name: _____

Units do not have to be included.

| **4 points each** | | |
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| 1 | A petri dish containing bacteria halves every 2 hours. If the dish contained 56 grams of bacteria, how many grams of bacteria will be left after 4 hours, in grams? | |
| 2 | Carl travels at 75 miles per hour, traveling from NANDville to NORville. Coming back, he travels at 50 miles per hour. These two cities are 100 miles apart. What is his average speed for the round trip, in miles per hour? | |
| 3 | Tickets for the play cost \$3 for members and \$7 for their guest. To cover expenses, they must sell \$4000 worth of tickets. If 423 member-tickets are sold, how many guest-tickets must be sold to cover expenses? | |
| 4 | To get from her house to the grocery store, Jane walks north along a straight path for 4 miles. She then turns right and walks east along a straight path for $\sqrt{48}$ miles. Find the bearing (or the angle E of N of the displacement) from Jane's house to the grocery store? | |
| 5 | The exponential decay equation for a sample of radioisotope with an initial count of 720 moles and a half-life of 5760 years is $$N = 720e^{-\frac{\ln 2}{5760}t}$$ Calculate the activity of the radioisotope after 2880 years. The activity is defined as the rate of radioactive disintegrations, $\frac{dN}{dt}$, in moles per second. Leave your result in simplest radical form. | |

| **5 points each** | | |
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| 6 | In 1930 the population of Waco, Texas was 630,000. The population increased to 1,430,000 by 1970, at a linear rate. Estimate its population by the year 2090. | |
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| 8 | A circular object with a radius of $\frac{1}{4}$ has a uniform chance of landing on any point of an 8 by 8 checkerboard. Each square on the board has a side length of 1. If the object is certain to land on the checkerboard with the possibility of overhang, find the probability that the object lands within one of the 64 squares on the checkerboard without crossing any line. | |
| 9 | Two doggy sleds are at the same location at $t = 0$ seconds. They start to move away from each other at an angle of 120 degrees. One doggy sled is moving at a rate of 3 m/s and the other at 5 m/s. Find the distance between the doggy sleds at $t = 2$ seconds, raised to the third power, in meters cubed. | |
| 10 | A heavy steel spherical ball bearing with a radius of 4 m strikes the surface of a large tub filled with a thick fluid to a depth of 8 m. The weight density of the fluid is $30000\frac{N}{m^3}$. It penetrates and sinks below the surface. How much work, expressed in Joules, is done ON the ball BY the buoyant force due to the fluid from the moment that the ball strikes the surface until it has just become completely submerged? *Work is defined as force multiplied by distance. In mathematical form* $W(h) = F(h) * h$. *Archimedes' principle states that the buoyant force is equal to the weight of the displaced fluid, which is represented by the function* $F(h) = 30000\,V(h)$. | |

| 6 points each | | |
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| 11 | On the Cheese moon, a chord between two craters measure 1500 km. The two craters subtend an angle of 90 degrees at the center of the Cheese moon. Given this information, calculate the surface area of the moon. | |
| 12 | For a math team party, Black Bolt orders a large, deep-dish, Chicago-style pizza that is 26" in diameter, 4" thick in the center and uniformly increasing to 6" of thickness at the edge! Medusa decides that she wants a large slice of pizza. Therefore, she cuts out a sector that is 60 degrees. What is the volume of the slice of pizza Medusa cut, in cubic inches? | |
| 13 | A rectangle has the vertices (0,5), (2,7), (5,0), and (7,2). If a solid were to be generated by revolving the rectangle about the y-axis, what would be its volume? | |
| 14 | Harley Quinn steals the Batmobile and drives it off a 36 ft cliff. The Batmobile's horizontal speed is given by the equation: $v_x = 48\frac{ft}{s}$ and its vertical position, in meters, is given by the equation $y(t) = -16t^2 + 32t + 36$. Determine the slope of the Batmobile's trajectory when it is 4 feet off the ground. | |
| 15 | A skier glides across a horizontal run. Suddenly, she encounters an abrupt slope change and begins to freefall through the air as she prepares to make a landing on the sloped hill. While she is in the air, her trajectory is given by the equation $$y = -\frac{1}{25}x^2 + 100$$ The equation that describes the slope of the hill is $y = -x + 100$. Calculate the vertical rate of position change (in ft/s) between the skier and the hill the moment she lands on the slope. Assume her initial velocity at the beginning of the flight is completely horizontal at $5\frac{ft}{s}$. | |

**2017 – 2018 Log1 Contest Round 2**

**Theta Applications – Answer Key**

Name: _____

Units do not have to be included.

| **4 points each** | | |
|---|---|---|
| 1 | A petri dish containing bacteria halves every 2 hours. If the dish contained 56 grams of bacteria, how many grams of bacteria will be left after 4 hours, in grams? | 14 |
| 2 | Carl travels at 75 miles per hour, traveling from NANDville to NORville. Coming back, he travels at 50 miles per hour. These two cities are 100 miles apart. What is his average speed for the round trip, in miles per hour? | 60 |
| 3 | Tickets for the play cost \$3 for members and \$7 for their guest. To cover expenses, they must sell \$4000 worth of tickets. If 423 member-tickets are sold, how many guest-tickets must be sold to cover expenses? | 391 |
| 4 | Nancy makes a circular pizza with an area of $4\pi$ ft$^2$. She cuts the pizza along two diameters perpendicular to each other, forming four slices of equal area. If the pizza is perfectly round, and each slice is exactly the same, find the perimeter of one slice, in feet. | $4+\pi$ |
| 5 | An airplane flies east for 150 km before turning 90° south and flying for 120 km. Find the shortest distance of the plane from its starting point, in km. | $30\sqrt{41}$ |

| **5 points each** | | |
|---|---|---|
| 6 | In 1930 the population of Waco, Texas was 630,000. The population increased to 1,430,000 by 1970, at a linear rate. Estimate its population by the year 2090. | 3,830,000 |
| 7 | Laura is going off to college and wants to make a budget for the first semester. She comes up with the following figures: Tuition - \$2207, Food - \$855, Books - \$565, Car - \$545, Rent - \$1243, and Miscellaneous - \$800. Since Laura is organized, she constructs a pie graph to show her budget. Determine the measure of the central angle for the sector labeled Rent. | $72°$ or $\frac{2}{5}\pi$ |
| 8 | A circular object with a radius of $\frac{1}{4}$ has a uniform chance of landing on any point of an 8 by 8 checkerboard. Each square on the board has a side length of 1. If the object is certain to land on the checkerboard with the possibility of overhang, find the probability that the object lands within one of the 64 squares on the checkerboard without crossing any line. | $\frac{1}{4}$ |
| 9 | Myrtle climbs to the top of the 25000-ft Mount Brie, the highest point on planet Cheddar Cheese. Ever the wise gal, she pulls out a super ball and tosses it off the top of the mountain. The ball bounces back up to 50% of its previous height with each successive bounce. Find the total vertical distance, in feet, that Myrtle's ball traveled if it continues bouncing with this pattern indefinitely. | 75000 |
| 10 | In the Snickers galaxy, there are 8 planets. On the first planet, there are 2,410 caramel people. On the second planet, 7,230 caramel people. On the third planet, 21,690 caramel people. If this pattern continues, how many total caramel people live in the galaxy? | 7,904,800 |

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| 11 | On the Cheese moon, a chord between two craters measure 1500 km. The two craters subtend an angle of 90 degrees at the center of the Cheese moon. Given this information, calculate the surface area of the moon. | $4500000\pi$ |
| 12 | For a math team party, Black Bolt orders a large, deep-dish, Chicago-style pizza that is 26" in diameter, 4" thick in the center and uniformly increasing to 6" of thickness at the edge! Medusa decides that she wants a large slice of pizza. Therefore, she cuts out a sector that is 60 degrees. What is the volume of the slice of pizza Medusa cut, in cubic inches? | $\frac{1352}{9}\pi$ |
| 13 | A rectangle has the vertices (0,5), (2,7), (5,0), and (7,2). If a solid were to be generated by revolving the rectangle about the y-axis, what would be its volume? | $140\pi$ |
| 14 | If Alex pulls a log with a force of 13-lbs at an angle of 60° with the horizontal and Kevin pulls the same log with a force of 16-lbs at an angle of 120° with the horizontal, what is the magnitude of the net force, in pounds, from Alex and Kevin? | $\sqrt{633}$ |
| 15 | A sugar cone with a height of 3 cm is filled with ice cream. Above the top surface of the cone, which has a diameter of 3 cm, the ice cream forms a hemispherical, concave downward dome also with a diameter of 3 cm. Calculate the volume of all the ice cream in and above the cone. | $\frac{9}{2}\pi$ |

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| 4 | To get from her house to the grocery store, Jane walks north along a straight path for 4 miles. She then turns right and walks east along a straight path for $\sqrt{48}$ miles. Find the bearing (or the angle E of N of the displacement) from Jane's house to the grocery store? | $\frac{\pi}{3}$ or $60°$ |
| 5 | An airplane flies east for 150 km before turning 90° south and flying for 120 km. Find the shortest distance of the plane from its starting point, in km. | $30\sqrt{41}$ |

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| 10 | In the Snickers galaxy, there are 8 planets. On the first planet, there are 2,410 caramel people. On the second planet, 7,230 caramel people. On the third planet, 21,690 caramel people. If this pattern continues, how many total caramel people live in the galaxy? | 7,904,800 |

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| 5 | The exponential decay equation for a sample of radioisotope with an initial count of 720 moles and a half-life of 5760 years is $$N = 720e^{-\frac{\ln 2}{5760}t}$$ Calculate the activity of the radioisotope after 2880 years. The activity is defined as the rate of radioactive disintegrations, $\frac{dN}{dt}$, in moles per second. Leave your result in simplest radical form. | $-\frac{\sqrt{2}}{16}\ln 2$ |

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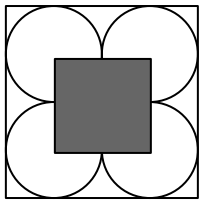
| 13 | A rectangle has the vertices (0,5), (2,7), (5,0), and (7,2). If a solid were to be generated by revolving the rectangle about the y-axis, what would be its volume? | $140\pi$ |
| 14 | Harley Quinn steals the Batmobile and drives it off a 36 ft cliff. The Batmobile's horizontal speed is given by the equation: $v_x = 48\frac{ft}{s}$ and its vertical position, in meters, is given by the equation $y(t) = -16t^2 + 32t + 36$. Determine the slope of the Batmobile's trajectory when it is 4 feet off the ground. | $-\frac{2}{3}\sqrt{3}$ |
| 15 | A skier glides across a horizontal run. Suddenly, she encounters an abrupt slope change and begins to freefall through the air as she prepares to make a landing on the sloped hill. While she is in the air, her trajectory is given by the equation $$y = -\frac{1}{25}x^2 + 100$$ The equation that describes the slope of the hill is $y = -x + 100$. Calculate the vertical rate of position change (in ft/s) between the skier and the hill the moment she lands on the slope. Assume her initial velocity at the beginning of the flight is completely horizontal at $5\frac{ft}{s}$. | $-5$ |

**2017 – 2018 Log1 Contest Round 2**
**Applications Solutions**

| Mu | Al | Th | Solution |
|---|---|---|---|
| 1 | 1 | 1 | Using the half-life formula,<br>$y = 56(0.5)^2$<br>$y = 56\,(0.25)$<br>$y = 14$ grams |
| 2 | 2 | 2 | Using the definition of harmonic mean:<br>$v = \frac{2}{\frac{1}{75}+\frac{1}{50}} = 60\frac{\text{mi}}{\text{hr}}$<br>Using physics principles:<br>$v = \frac{\text{distance}}{\text{time}} = \frac{200 \text{ miles}}{\frac{10}{3} \text{ hours}} = 60\frac{\text{mi}}{\text{hr}}$ |
| 3 | 3 | 3 | 423 @ \$3 per student tickets adds up to \$1269, leaving \$2731 left over to sell. To cover their expenses, they then need to sell<br>$\frac{\$2731}{7} = 391$ guest tickets |
| 4 | 4 | | The two paths form a right triangle. The side opposite the bearing angle is $\sqrt{48}$ miles and the side adjacent to it is 4 miles. The bearing angle is defined by the inverse tangent.<br>$\theta = \tan^{-1}\frac{\sqrt{48}}{4} = \tan^{-1}\frac{4\sqrt{3}}{4} = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$ or $60°$ |
| | | 4 | The radius of the pizza is 2 ft. It is divided into quarters each of equal area. The arc length of the "rounded edge" of each slice is ¼ the circumference of the whole pizza. The circumference of the pizza is<br>$C = 2\pi r = 4\pi$ ft.<br>The arc length is just $\pi$ ft.<br><br>The perimeter of a single slice would be<br>$C = 2 + 2 + \pi = 4 + \pi$ ft |

| | | | |
|---|---|---|---|
| 5 | | | $$\frac{dN}{dt} = \frac{720(-\ln 2)}{5760} e^{\frac{-\ln 2}{5760}t}$$ Evaluate at $t = 2880$ years. $$\frac{dN}{dt} = \frac{-\ln 2}{8} e^{-\frac{\ln 2}{5760}(2880)}$$ $$\frac{dN}{dt} = -\frac{\ln 2}{8} e^{-\frac{\ln 2}{2}} = -\frac{\ln 2}{8} e^{-\frac{\ln 2}{2}} = -\frac{\ln 2}{8}\left(\frac{1}{2}\right)^{\frac{1}{2}}$$ $$\frac{dN}{dt} = -\frac{\ln 2}{2^3}\left(\frac{1}{\sqrt{2}}\right) = -\frac{\ln 2}{8\sqrt{2}} = -\frac{\sqrt{2}}{16}\ln 2$$ |
| | 5 | 5 | Use the Pythagorean Theorem to find the hypotenuse of this right triangle. $$c^2 = 150^2 + 120^2 = 36900$$ $$c = 30\sqrt{41}\text{ km}$$ |
| 6 | 6 | 6 | Determine the equation for this linear trend. $$y = mx + b$$ $$m = \frac{\Delta y}{\Delta x}$$ $$\Delta x = 1970 - 1930 = 40$$ $$\Delta y = 1{,}430{,}000 - 630{,}000 = 800{,}000$$ $b = 630{,}000$ when $x = 1930$ $$m = \frac{800{,}000}{40} = 20{,}000$$ $$y = 20000(x - 1930) + 630000$$ $$y = 20000(2090 - 1930) + 630000$$ $$y = 3{,}830{,}000$$ |
| 7 | 7 | 7 | Add up all costs to determine the total budget. $$\text{Total budget} = \$6215$$ The sector for Rent occupies 1/5 of the total area of the pie graph. $$A_{\text{Rent}} = \frac{1243}{6215} = \frac{1}{5}$$ A sector that is 1/5 of the whole pie graph has an arc central angle that subtends 1/5 of the whole circle. $$\theta_{\text{Rent}} = \frac{1}{5} * \frac{360}{1} = 72° \text{ or } \frac{2}{5}\pi$$ |
| 8 | 8 | 8 | The center of the circular object can land anywhere within any gray square as shown above. The total area of this gray square would be 1/4 since each side is 1/2. If one just considers a single square, the probability of landing within this square is $\frac{\frac{1}{4}}{1} = \frac{1}{4}$. Since every square on the checkerboard is identical, the representative areas increase proportionally keeping the probability the same, even if the checker board were infinite in size.  |

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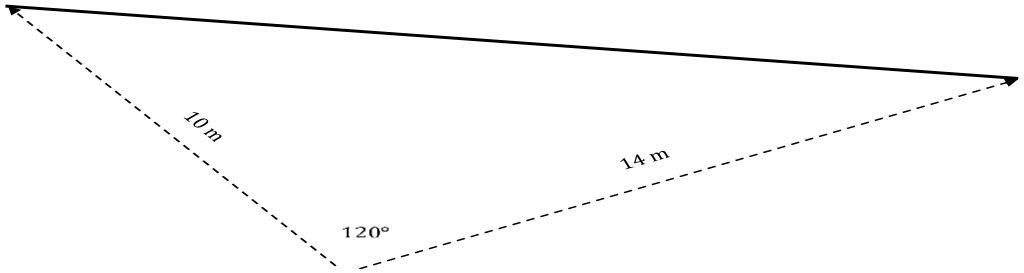
| 9 | 9 | | At 2 seconds, one tractor has moved 10 meters and the other has moved 14 meters. The distance between the two sleds is the third side of the triangle formed by these two displacement vectors.<br>  <br><br>Using the Law of Cosines, we have the equation:<br>$c^2 = 6^2 + 10^2 - 2(6)(10)\cos(120).$<br>$c^2 = 136 - 120\left(-\frac{1}{2}\right)$<br>$c^2 = 196 \rightarrow c = 14$<br>$c^3 = 14^3 = 2744\ m^3$ |
| | | 9 | Solve for the converging, infinite series for the downward motions first.<br>$S_{Down} = 25000 + 12500 + 6125 + \cdots.$<br>$S_{\infty} = \frac{a}{1-r} = \frac{25000}{1-0.5} = 50000$<br>$S_{Up} = 12500 + 6125 + \cdots.$<br>$S_{Up} = \frac{a}{1-r} = \frac{12500}{1-0.5} = 25000$<br>$S_{Total} = 75000$ |

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| 10 | | | The work differential is, $dW = F(h)dh$. We must derive the formula for Force as a function of depth that the ball is in the thick fluid. According to Archimedes' principle,<br>$$F_B = 30000V_B$$<br>However, as the sphere submerges, the displaced volume changes so we setup the buoyant force differential.<br>$$dF = 30000dV$$<br>The equation for the circle being revolved to produce the sphere will be $x^2 + (h-4)^2 = 16$. Solving for $x^2$, $x^2 = 8h - h^2$ Integrating….<br>$$F(h) = \int_0^h 30000\pi x^2 dh = 30000\pi \int_0^h (8h - h^2)dh$$<br>$$F(h) = 30000\pi\left(4h^2 - \frac{1}{3}h^3 + C\right)$$<br>The constant of integration must be $C = 0$ since the buoyant force would have to be 0 when the ball is not submerged. Integrate again to obtain the work<br>$$W = 30000\pi \int_0^8 \left(4h^2 - \frac{1}{3}h^3\right) dh$$<br>$$W = 30000\pi\left(\frac{4}{3}h^3 - \frac{1}{12}h^4\right)\Big|_0^8$$<br>$$W = 30000\pi\left(\frac{4}{3}8^3 - \frac{1}{12}8^4\right)$$<br>$$W = 30000\pi\left(\frac{2^{12}}{6} - \frac{2^{11}}{6}\right) = 30000\pi\left(\frac{2^{11}}{6}\right) = 5000\pi(2^{11})$$<br>$$W = 5000\pi(2048) = 10240000\pi \text{ Joules}$$ |
| | 10 | 10 | On each planet, the amount of caramel people is tripled from the last planet, starting at 2,410 on the first planet.<br>$$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$$<br>$$S_8 = 2410\frac{(1 - 3^8)}{1 - 3} = 2410\frac{1 - 6561}{1 - 3}$$<br>$$S_8 = 2410(3280) = 7{,}904{,}800$$ |
| 11 | 11 | 11 | The chord formed by connecting the two craters with a line segment 1500 km long forms the third side of an isosceles right triangle with its legs being the radii of the moon. Bisecting the angle forms two 45° right triangles with the side opposite the angle equal to 750 km. Using the rules for 45-45-90 right triangles, the length of each leg of the triangle would be $\frac{1500}{\sqrt{2}}$<br><br>The surface area of a sphere is<br>$$A = 4\pi r^2 = 4\pi\left(\frac{1500}{\sqrt{2}}\right)^2 = 4\pi\left(\frac{2250000}{2}\right) = 4500000\pi$$ |

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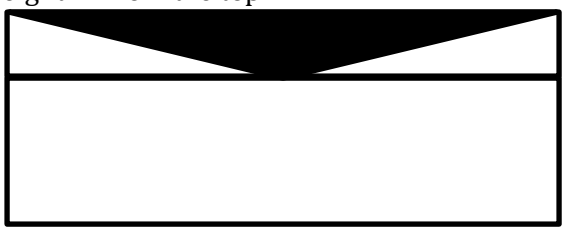
| 12 | 12 | 12 | The volume of the pizza may be calculated by considering the pizza as a cylinder with a radius of 13" and a height of 6" and then removing an upside-down cone of radius 13" and height 2" from the top.<br>  <br>Calculate the volume of the cylinder.<br>$$V_{Cylinder} = \pi(13^2)(6) = 169(6)\pi = 1014\pi \text{ in}^3$$<br>The volume of the cone is<br>$$V_{Cone} = \frac{1}{3}\pi(13^2)(2) = \frac{338}{3}\pi \text{ in}^3$$<br>The volume of the pizza is<br>$$V_{Pizza} = V_{Cylinder} - V_{Cone} = \left(1014 - \frac{338}{3}\right)\pi$$<br>$$V_{Pizza} = \frac{2704}{3}\pi = \frac{1352}{9}\pi \text{ in}^3$$ |
| 13 | 13 | 13 | **Method 1: Mu, Alpha and Theta**<br>The vertices construct a rectangle. The edge of the rectangle formed by the vertices (2,7) and (7,2) is collinear with the line $y = -x + 9$. Revolve $y = -x + 9$ about the x-axis and a cone with radius 9 and height 9 is generated. Its volume is 243π. Sub-divide the cone into 4 regions. One of the regions is rectangular, as presented in the problem. The other three regions are triangular.<br>Triangle A: (0,5), (0,9), and (2,7)<br>Triangle B: (5,0), (9,0), and (7,2)<br>Triangle C: (0,0), (0,5), and (5,0)<br>Revolving Triangle C results in a cone, $V_C = \frac{1}{3}\pi(5^2)5 = \frac{125}{3}\pi$<br><br>Revolving Triangle B results in a triangular prism with cross-sectional area $A_B = ½(4)(2) = 4$ and length $L_B = 2\pi(7) = 14\pi$. $V_B = A_B L_B = (4)14\pi = 56\pi$<br><br>Revolving Triangle A results in prism that consists of two cones, base-to-base, each with a radius of 2 and height of 2. The volume of this prism is $V_A = \frac{2}{3}\pi(2^2)2 = \frac{16}{3}\pi$<br>Add the volumes of triangles A, B, and C. $V_{ABC} = 103\pi$<br>Volume of the rectangular region is $V = (243 - 103)\pi = 140\pi$<br>**Method 2: Mu**<br>The shape is bounded by 4 equations<br>$$y_1 = x + 5, y_2 = 9 - x, y_3 = x - 5, y_4 = 5 - x$$<br>Divide the shape into two layers: $y_A = 0 \to 2, y_B = 2 \to 7$<br>Integrate using the Washer Method<br>$$V_A = \pi\int_0^2 (y_3 - y_4)dx = \pi\int_0^2 20ydy = 40\pi$$<br>$$V_B = \pi\int_2^7 (y_2 - y_3)dx = \pi\int_2^7 (56 - 8y)dy = 100\pi$$<br>$$V_T = V_A + V_B = 140\pi$$ |

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| 14 | | | The Batmobile's velocity equation is given as $v_x = \frac{dx}{dt} = 48\frac{ft}{s}$. Differentiate its vertical position and divide through by dx.<br>$$dy = -32t\,dt + 32\,dt$$<br>$$\frac{dy}{dx} = -32t\frac{dt}{dx} + 32\frac{dt}{dx} = -32(t-1)\frac{dt}{dx}$$<br>Solve for $y(t) = 4$.<br>$$4 = -16t^2 + 32t + 36$$<br>$$0 = -16t^2 + 32t + 32$$<br>$$0 = -16(t^2 - 2t - 2)$$<br>$$0 = t^2 - 2t - 2$$<br>$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$<br>$$t = 1 \pm \frac{1}{2}\sqrt{12} = 1 \pm \sqrt{3};\ \text{Choose } 1 + \sqrt{3}$$<br>$$\frac{dy}{dx} = -32\left(1 + \sqrt{3} - 1\right)\left(\frac{1}{48}\right) = -\frac{32}{48}\sqrt{3} = -\frac{2}{3}\sqrt{3}$$ |
| | 14 | 14 | Add the two force vectors to obtain the net force. The net force forms a triangle with the unknown side equal to the net force. The angle included by Alex's and Kevin's forces is 120 degrees.<br>$$c^2 = 13^2 + 16^2 - 2(13)(16)\cos 120$$<br>$$c = \sqrt{633}\ \text{lbs}$$ |

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| 15 | | | The difference in vertical position between the skier and the ski hill for any value of x is<br>$$h = -\frac{1}{25}x^2 + 100 - (-x + 100) = -\frac{1}{25}x^2 + x$$<br>$$\frac{dh}{dt} = -\frac{2}{25}x\frac{dx}{dt} + \frac{dx}{dt}$$<br>To evaluate, one must know the intersection points of the two equations for the trajectory and the hill. In other words, find the roots of the equation $h = -\frac{1}{25}x^2 + x$<br>$$0 = -\frac{1}{25}x^2 + x \rightarrow x^2 - 25x = 0$$<br>$$x(x - 25) = 0$$<br>The roots are $x = 0$ ft and $x = 25$ ft<br>Choose $x = 25$ ft.<br>The rate of change of vertical separation distance is<br>$$\frac{dh}{dt} = -\frac{2}{25}x\frac{dx}{dt} + \frac{dx}{dt} = -\frac{2}{25}(25)(5) + 5 = -5\frac{ft}{s}$$ |
| | 15 | 15 | $$\frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\right)\pi r^3$$<br>$$\frac{1}{3}\pi\left(\frac{3}{2}\right)^2(3) + \frac{1}{2}\left(\frac{4}{3}\right)\pi\left(\frac{3}{2}\right)^3$$<br>$$\frac{9}{2}\pi$$ |