

2017 – 2018 Log1 Contest Round 1

Theta Functions

Name: _____

Units do not have to be included.

4 points each		
1	If $h(x) = 3x + 2$, find the value of x when $h(x) = 8$.	
2	The function for the volume of a right cylindrical cone is $V(h) = \frac{1}{3}\pi r^2 h$. Calculate the height of this cone if its volume is $\frac{32}{3}\pi$ and its radius is 2.	
3	If $g(x) = 5x - 3$ and $k(x) = 2x$, evaluate $g(k(2))$	
4	Let $f(x) = 3x^2 - 2$ and $g(x) = 4x + 1$. Determine the value of $h(2)$ if the function $h(x) = \sqrt{f(x)^{g(x)}}$. Reduce the answer into the simplest root form. Note that $g(x)$ is an exponent of $f(x)$.	
5	Let $f(x) = x^3 - 2x^2 - x + 2$. If $(x + 1)$ is a factor of $f(x)$, then find the sum of all the roots.	

5 points each		
6	<p>Kate throws a ball off a 126 foot cliff with an initial upward velocity of 8 feet/sec. The height of the ball t seconds after it is thrown is given by the function</p> $h(t) = -16t^2 + 8t + 126$ <p>Find the maximum height of the ball in feet.</p>	
7	Given two numbers, a and b , determine the sum of their cubes if $a + b = 3$ and $a^2 + b^2 = 6$. In other words, calculate $a^3 + b^3$.	
8	<p>Consider the 2 functions shown below.</p> $f(x) = 12x^2 - 8x + 18$ $g(x) = 5 \log_{10} 2x$ <p>Evaluate $f(g(f(2)))$</p>	
9	<p>Determine the equation(s) for the vertical asymptote(s) of the function</p> $f(x) = \frac{2x^2 + x - 15}{x^2 - 3x - 4}$	
10	Find the minimum value of $f(x) = 10x^2 - 2x - 5$	

6 points each

11	If $f(x) = \frac{x+6}{x-5}$, find $f^{-1}(x)$	
12	Let $f(x)$ be a function as expressed below. $f(x) = \frac{x+4}{x-2}$ Determine the expression for $f(f(f(x)))$.	
13	Find the coefficient of the 7th-degree term in the expansion of $(3x + 2)^{11}$. You may express your answer as a product of positive factors if you desire.	
14	Find all the roots of the function $f(x) = x^3 + 5x^2 + 9x + 5$	
15	Let $f(x) = x^3 + 11x^2 + 3x - 135$. If the three roots of $f(x)$ are a , b , and c , what is the maximum value of $a - b + c$.	

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4	Compute the limit for the following function. $\lim_{x \rightarrow 0} \frac{x^2 + 6x}{x}$	
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9	Let $f(x) = (\sin x + \cos x)^2$. Evaluate the inverse function $f^{-1}\left(\frac{3}{2}\right)$ on the domain $\left[0, \frac{\pi}{2}\right]$.	
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13	Find the coefficient of the 7th-degree term in the expansion of $(3x + 2)^{11}$. You may express your answer as a product of positive factors if you desire.	
14	Suppose $f(x) = e^x$, $g(x) = a \ln x$, and $h(x) = be^x$ Let $h(1) = a$ and $f \circ g \circ h(1) = e^b$ Calculate the exact values for $\ln a$ and $\ln b$.	
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4	Compute the limit for the following function. $\lim_{x \rightarrow 0} \frac{x^2 + 6x}{x}$	
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9	Let $f(x) = (\sin x + \cos x)^2$. Evaluate the inverse function $f^{-1}(\frac{3}{2})$ on the domain $[0, \frac{\pi}{2}]$.	
10	Let $f(x) = \sin(6x) \cos(3x)$ and $g(x) = \frac{1}{3}x$ Evaluate the definite integral shown below. $\int_0^{\frac{\pi}{2}} (f \circ g(x)) dx$	

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15	Let $f(x) = \cos x \tan^3 x$ and $g(x) = \int f(x) dx$. If $g(0) = 2$ and $h(x) = g^{-1}(x)$, determine all possible values of $h\left(-\frac{5}{2}\right)$ within the interval $[0, 2\pi]$.	

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2	The function for the volume of a right cylindrical cone is $V(h) = \frac{1}{3}\pi r^2 h$. Calculate the height of this cone if its volume is $\frac{32}{3}\pi$ and its radius is 2.	8
3	If $g(x) = 5x - 3$ and $k(x) = 2x$, evaluate $g(k(2))$	17
4	Let $f(x) = 3x^2 - 2$ and $g(x) = 4x + 1$. Determine the value of $h(2)$ if the function $h(x) = \sqrt{f(x)^{g(x)}}$. Reduce the answer into the simplest root form. Note that $g(x)$ is an exponent of $f(x)$.	$10000\sqrt{10}$
5	Let $f(x) = x^3 - 2x^2 - x + 2$. If $(x + 1)$ is a factor of $f(x)$, then find the sum of all the roots.	2

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6	Kate throws a ball off a 126 foot cliff with an initial upward velocity of 8 feet/sec. The height of the ball t seconds after it is thrown is given by the function $h(t) = -16t^2 + 8t + 126$ Find the maximum height of the ball in feet.	127
7	Given two numbers, a and b , determine the sum of their cubes if $a + b = 3$ and $a^2 + b^2 = 6$. In other words, calculate $a^3 + b^3$.	$\frac{27}{2}$
8	Consider the 2 functions shown below. $f(x) = 12x^2 - 8x + 18$ $g(x) = 5 \log_{10} 2x$ Evaluate $f(g(f(2)))$	1138
9	Determine the equation(s) for the vertical asymptote(s) of the function $f(x) = \frac{2x^2 + x - 15}{x^2 - 3x - 4}$	$x = 4$ $x = -1$
10	Find the minimum value of $f(x) = 10x^2 - 2x - 5$	$-\frac{51}{10}$ or -5.1

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11	If $f(x) = \frac{x+6}{x-5}$, find $f^{-1}(x)$	$\frac{5x+6}{x-1}$
12	Let $f(x)$ be a function as expressed below. $f(x) = \frac{x+4}{x-2}$ Determine the expression for $f(f(f(x)))$.	$\frac{x+28}{7x-20}$
13	Find the coefficient of the 7th-degree term in the expansion of $(3x+2)^{11}$. You may express your answer as a product of positive factors if you desire.	$2^5 3^8 5 * 11$ Or 11,547,360
14	Find all the roots of the function $f(x) = x^3 + 5x^2 + 9x + 5$	$x = -1$ $x = -2 + i$ $x = -2 - i$
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9	Let $f(x) = (\sin x + \cos x)^2$. Evaluate the inverse function $f^{-1}(\frac{3}{2})$ on the domain $[0, \frac{\pi}{2}]$.	$\frac{\pi}{12}, \frac{5\pi}{12}$
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5	Given the function $f(x) = x^2 + 3e^{3x} + 2$, evaluate $f'(\ln 2)$.	$2 \ln 2 + 72$ Or $\ln 4 + 72$

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10	Let $f(x) = \sin(6x) \cos(3x)$ and $g(x) = \frac{1}{3}x$ Evaluate the definite integral shown below. $\int_0^{\frac{\pi}{2}} (f \circ g(x)) dx$	$\frac{2}{3}$

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Functions Solutions

Mu	Al	Th	Solution
1	1	1	$8 = 3x + 2$ $3x = 6$ $x = 2$
2	2	2	<p>Set $V(h) = \frac{32}{3}\pi$. Solve for h.</p> $\frac{32}{3}\pi = \frac{1}{3}\pi 2^2 h$ $h = 8$
3	3	3	$k(2) = 2(2) = 4$ $g(k(2)) = g(4) = 5(4) - 3 = 17$
4	4		$\lim_{x \rightarrow 0} \frac{x^2 + 6x}{x} = \lim_{x \rightarrow 0} \frac{x(x+6)}{x} = \lim_{x \rightarrow 0} x + 6 = 6$
		4	$h(2) = \sqrt{10^9} = \sqrt{10^8 * 10} = 10^4 \sqrt{10} = 10000\sqrt{10}$
5			$f'(x) = 2x + 9e^{3x}$ $f'(\ln 2) = 2 \ln 2 + 9e^{3(\ln 2)}$ $f'(\ln 2) = 2 \ln 2 + 9(e^{\ln 2})^3$ $f'(\ln 2) = 2 \ln 2 + 9 * 2^3 = 2 \ln 2 + 72$
	5	5	<p>Factoring out $(x + 1)$ from $f(x)$ results in</p> $f(x) = (x + 1)(x^2 - 3x + 2)$ $f(x) = (x + 1)(x - 1)(x - 2)$ <p>The three roots are -1, 1 and 2. The sum of all three roots is 2</p> <p>Alternatively, the sum of the roots for a cubic has the same form as that for a quadratic, $-\frac{b}{a}$ when</p> $f(x) = ax^3 + bx^2 + cx + d$

6	6	6	<p>The vertex is defined at the point (h, k) when the equation for the parabola is written in vertex form.</p> $h = -\frac{b}{2a} = -\frac{8}{-32} = \frac{1}{4}$ <p>The value at k would be the maximum height.</p> $k = f(0.25) = -16(0.25^2) + 8(0.25) + 126$ $k = 127$ <p>Starting from basic principles. The equation in vertex form is $4p(y - k) = (x - h)^2$. Expand.</p> $y - k = \frac{1}{4p}x^2 - \frac{h}{2p}x - \frac{h^2}{4p}$ $y = \frac{1}{4p}x^2 - \frac{h}{2p}x - \frac{h^2}{4p} + k$ <p>Thus,</p> $\frac{1}{4p} = -16 \rightarrow p = -\frac{1}{64}$ $-\frac{h}{2p} = 8 \rightarrow h = \frac{1}{4}$ $k - \frac{h^2}{4p} = 126 \rightarrow k = 126 + 1 = 127$
7	7	7	<p>Expand $(a + b)^3 = 3^3 = 27$</p> $a^3 + 3a^2b + 3ab^2 + b^3 = 27$ <p>Factor out 3ab.</p> $a^3 + 3ab(a + b) + b^3 = 27$ $a^3 + b^3 = 27 - 3ab(a + b) = 27 - 9ab$ <p>Evaluate ab.</p> $(a + b)^2 = a^2 + 2ab + b^2 = 3^2 = 9$ $ab = \frac{9 - (a^2 + b^2)}{2} = \frac{9 - 6}{2} = \frac{3}{2}$ $a^3 + b^3 = 27 - 9 * \left(\frac{3}{2}\right) = \frac{27}{2}$
8	8	8	$f(2) = 12(2^2) - 8(2) + 18 = 50$ $g(50) = 5 \log_{10} 2(50) = 5 \log_{10} 100 = 5(2) = 10$ $f(10) = 12(10)^2 - 8(10) + 18 = 1200 - 80 + 18$ $f(10) = 1138$

9	9	<p>Set $f(x) = \frac{3}{2}$ and solve for x</p> $\frac{3}{2} = (\sin x + \cos x)^2$ $\frac{3}{2} = 1 + \sin 2x$ $\sin 2x = \frac{1}{2}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}$
	9	<p>Factor the numerator and denominator:</p> $f(x) = \frac{(x+3)(2x-5)}{(x+1)(x-4)}$ <p>Vertical asymptote at $x = 4$ and $x = -1$</p>
10		$f \circ g(x) = \sin\left(6\left(\frac{1}{3}x\right)\right) \cos\left(3\left(\frac{1}{3}x\right)\right)$ $f \circ g(x) = \sin(2x) \cos(x)$ $f \circ g(x) = 2 \sin x \cos^2 x$ $\int_0^{\frac{\pi}{2}} (f \circ g(x)) dx = \int_0^{\frac{\pi}{2}} 2 \sin x \cos^2 x dx$ $u = \cos x \quad du = -\sin x dx$ $-2du = 2 \sin x dx$ $\int_1^0 -2u^2 du = -\frac{2}{3} 0^3 - \left(-\frac{2}{3}(1)^3\right) = \frac{2}{3}$
10	10	<p>The x-coordinate of the parabola's vertex is located at</p> $x = -\frac{b}{2a} = -\frac{-2}{2(10)} = \frac{1}{10}$ <p>Since the parabola is concave upward, evaluate $f\left(\frac{1}{10}\right)$.</p> $f\left(\frac{1}{10}\right) = 10\left(\frac{1}{10}\right)^2 - 2\left(\frac{1}{10}\right) - 5$ $f_{\min} = \frac{10}{100} - \frac{20}{100} - \frac{500}{100}$ $f_{\min} = -\frac{510}{100} = -\frac{51}{10}$

11	11	11	<p>Setup the inverse function in the following way.</p> $x = \frac{y + 6}{y - 5}$ <p>Solve for y.</p> $x(y - 5) = y + 6 \rightarrow yx - 5x = y + 6$ $yx - y = 5x + 6$ $y(x - 1) = 5x + 6$ $f^{-1}(x) = \frac{5x + 6}{x - 1}$
12	12	12	$f(f(x)) = \frac{\frac{x+4}{x-2} + 4}{\frac{x+4}{x-2} - 2} = \frac{x+4+4x-8}{x+4-2x+4} = \frac{5x-4}{-x+8}$ $f(f(f(x))) = \frac{\left(\frac{5x-4}{-x+8}\right) + 4}{\frac{5x-4}{-x+8} - 2} = \frac{5x-4-4x+32}{5x-4+2x-16}$ $f(f(f(x))) = \frac{x+28}{7x-20}$
13	13	13	<p>The binomial expansion theorem gives the 7th degree term as</p> $\binom{11}{4} (3x)^7 (2)^4$ <p>Evaluating:</p> $\frac{11 * 10 * 9 * 8}{4 * 3 * 2} * 3^7 * 2^4$ $2^4 * 3^7 * 3 * 10 * 11 = 11,547,360$
14	14		<p>Since $h(1) = a$, it follows that $a = be$. Evaluate the composite function $f \circ g \circ h(1)$.</p> $g \circ h(1) = g(a) = a \ln(be)$ $f \circ g \circ h(1) = f(a \ln(be)) = e^{a \ln(be)} = e^b$ $b = a \ln(be)$ $b = be \ln(be)$ $1 = e(\ln b + \ln(e)) = e(\ln b + 1)$ $\ln b = e^{-1} - 1$ $\ln a = \ln(be) = \ln b + \ln e = \ln b + 1$ $\ln a = e^{-1} - 1 + 1$ $\ln a = e^{-1}$

	14	<p>A factor could potentially be $(x \pm p)$ where p are the possible factors of 5.</p> <p>It turns out that $(x + 1)$ is a factor.</p> $f(x) = (x + 1)(x^2 + 4x + 5)$ <p>Setting to 0:</p> $0 = (x + 1)(x^2 + 4x + 5)$ <p>One root of the function is</p> $x = -1$ <p>The other roots are found from the quadratic factor.</p> $\frac{-4 \pm \sqrt{16 - 4(1)(5)}}{2(1)} = -2 \pm \frac{\sqrt{-4}}{2} = -2 \pm i$
15	15	<p>The function $f(x)$ can be written as</p> $f(x) = (x + 9)(x - 3)(x + 5)$ <p>Thus, the roots are -9, 3, and -5.</p> <p>To maximize $a - b + c$, we need to make b the smallest root. Hence,</p> $a - b + c = 3 - (-9) - 5 = 7$

Integrate $f(x)$

$$g(x) = \int f(x) dx = g(x) = \int \cos x \tan^3 x dx$$

$$g(x) = \int \cos x \tan x \tan^2 x dx$$

$$g(x) = \int \sin x (\sec^2 x - 1) dx$$

$$g(x) = \int \sin x (\cos x)^{-2} dx - \int \sin x dx$$

$$u = \cos x \quad du = -\sin x dx$$

$$g(x) = \int -u^{-2} du - \int \sin x dx$$

$$g(x) = u^{-1} + \cos x + C$$

$$g(x) = (\cos x)^{-1} + \cos x + C$$

$$g(0) = (1)^{-1} + 1 + C = 2$$

$$C = 0$$

Express $g^{-1}(x)$ as $x = (\cos y)^{-1} + \cos y$

$$-\frac{5}{2} = (\cos y)^{-1} + \cos y$$

$$-\frac{5}{2} = \frac{\cos^2 y + 1}{\cos y}$$

$$-\frac{5}{2} \cos y = \cos^2 y + 1 \rightarrow 0 = 2 \cos^2 y + 5 \cos y + 2$$

$$\cos y = \frac{[-5 \pm \sqrt{25 - 4(2)(2)}]}{2 * 2}$$

$$\cos y = -\frac{5}{4} \pm \frac{3}{4}; \cos y = (-2, -\frac{1}{2})$$

$$y = h\left(-\frac{5}{2}\right) = \left(\frac{2}{3}\pi, \frac{4}{3}\pi\right)$$