2016 – 2017 Log1 Contest Round 3

Theta Individual

Name: _____

Units do not have to be included.

4 points each		
1	Convert the base 10 number 47 into a base 2 number.	
2	Two times the degree measure of the supplement of an angle exceeds the sum of the degree measures of the angle and one-fifth of the complement of the angle by 202°. Find the degree measure of the complement of this angle.	
3	One-sixth of one-seventh of one-eighth of one-ninth of one-tenth of a number is 5. What is the second most significant digit (2nd digit from the left) in the number?	
4	Write in simplest radical form: $\left(\left(\sqrt[4]{\sqrt[10]{6}}\right)^2\right)^{15}$	
5	Find the sum of $\sum_{k=7}^{\infty} 5\left(\frac{2}{5}\right)^{k-1}$ and leave in the form $\frac{a}{b}$.	

5 points each		
6	After a binomial expansion of the expression $\left(a+\frac{1}{a}\right)^{12}$, what is the value of the constant term?	
7	If the number 92,523,1N6 is divisible by 4, then what is the sum of possible values for N?	
8	Nine people enter a room and sit at random in a row of nine chairs. In how many ways can the best friends Izaya, Truman, and Colleen sit together in the row?	
9	Let the operation # be defined by $a\#b = a^2 + 5b$. What is the value of (3#4) # (5#6)?	
10	Find the area of the circle that circumscribes an equilateral triangle with a side of 48. Leave your answer in terms of pi.	

6 points each		
11	A silver bag contains 5 chartreuse and 3 magenta buttons. A gold bag contains 3 chartreuse and 2 magenta buttons. One button is randomly selected from the silver bag and its color is noted. If it is chartreuse, 4 chartreuse buttons are added to the gold bag. If it is magenta, 6 magentas are added to the gold bag. A button is then selected from the gold bag. What are the chances that the button selected from the gold is magenta?	
12	Truman and Colin regularly walk on a 2-mile trail that has one single entry point. To complete the trail, they must walk a round trip. If both randomly choose a time on the hour between 8 AM and 12 Noon and they both walk at 2 mph, what is the probability that their paths will intersect?	
13	Sarah is getting paid today at work. She didn't notice it, but her boss accidentally switched the number of dollars and cents when he paid her. Later, her friend gave her 65 cents to buy a cupcake. Sarah was shocked when she realized she only had half the value of her usual monthly payment. How much money was her boss supposed to give her?	
14	Consider the sequence, $2!, 2!+3!, 2!+3!+4!, \ldots, 2!+3!+4!+\cdots 20!$, where n! is n factorial. How many perfect squares are in this sequence?	
15	What is the product of the solutions for the logarithmic shown below? $$\log_{729} x - \log_x 27 = \frac{1}{2}$$	

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4	The cafeteria has three vending machines. On any given day vending machine X has a 20% chance of being out of order, vending machine Y has a 25% chance of being out of order, and vending machine Z has a 50% chance of working perfectly fine. Determine the probability that on any given day all machines won't be broken.	
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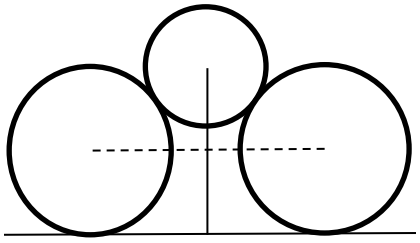
9	A small circle of radius, $r = 3$ cm, rests on top of two larger, identical circles of radius $R = 5$ cm. The two larger circles rest on a flat horizontal surface. Their centers are separated by a distance 12 cm. Determine the distance, H, from the center of the small circle to the horizontal surface. In other words, find the length of the solid vertical line shown in the diagram below. State your answer in the form $a + b\sqrt{c}$ 	
10	Find the area of the circle that circumscribes an equilateral triangle with a side of 48. Leave your answer in terms of pi.	

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14	A girl holds a large bubble ring by a handle. Strangely, this bubble ring is not circular but rather, it is elliptical. The equation $4x^2 + 9y^2 = 36$ models the shape of the bubble ring, in units of feet. The girl creates a helical tube that rises 4 feet vertically as she rotates the ring through 360 degrees. Calculate the volume of this elliptical tube with a helical radius of 2 feet.	
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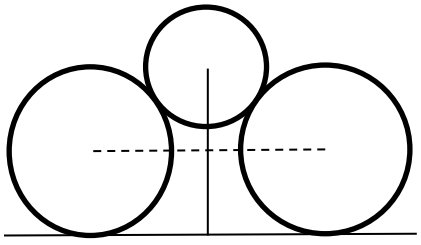
9	A small circle of radius, $r = 3$ cm, rests on top of two larger, identical circles of radius $R = 5$ cm. The two larger circles rest on a flat horizontal surface. Their centers are separated by a distance 12 cm. Determine the distance, H, from the center of the small circle to the horizontal surface. In other words, find the length of the solid vertical line shown in the diagram below. State your answer in the form $a + b\sqrt{c}$ 	
10	There is a bright light source that is set at ground level and located 50 m from the foot of a house. The light source and the house are on horizontal ground. Truman is 1.6 m tall and hops in a straight line towards the house at $0.5\frac{m}{s}$. His shadow is cast upon the vertical wall of the house. How fast, in m/s, is Truman's shadow on the house shortening at the instant when he is at 30 m from the house? Express your answer as a fraction in simplest form.	

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15	Evaluate $\int_{\sqrt{2}}^{\sqrt{3}} \frac{3x}{(x^2-2)+\sqrt{x^2-2}}\,dx$	

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3	One-sixth of one-seventh of one-eighth of one-ninth of one-tenth of a number is 5. What is the second most significant digit (2nd digit from the left) in the number?	5
4	Write in simplest radical form: $\left(\left(\sqrt[4]{\sqrt[10]{6}}\right)^2\right)^{15}$	$\sqrt[4]{216}$
5	Find the sum of $\sum_{k=7}^{\infty} 5\left(\frac{2}{5}\right)^{k-1}$ and leave in the form $\frac{a}{b}$.	$\frac{64}{1875}$

5 points each		
6	After a binomial expansion of the expression $\left(a+\frac{1}{a}\right)^{12}$, what is the value of the constant term?	924
7	If the number 92,523,1N6 is divisible by 4, then what is the sum of possible values for N?	25
8	Nine people enter a room and sit at random in a row of nine chairs. In how many ways can the best friends Izaya, Truman, and Colleen sit together in the row?	30240
9	Let the operation # be defined by $a\#b = a^2 + 5b$. What is the value of (3#4) # (5#6)?	1116
10	Find the area of the circle that circumscribes an equilateral triangle with a side of 48. Leave your answer in terms of pi.	768π

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12	Truman and Colin regularly walk on a 2-mile trail that has one single entry point. To complete the trail, they must walk a round trip. If both randomly choose a time on the hour between 8 AM and 12 Noon and they both walk at 2 mph, what is the probability that their paths will intersect?	$\frac{19}{25}$
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14	Consider the sequence, $2!, 2! + 3!, 2! + 3! + 4!, \ldots, 2! + 3! + 4! + \cdots 20!$, where n! is n factorial. How many perfect squares are in this sequence?	0
15	What is the product of the solutions for the logarithmic shown below? $$\log_{729} x - \log_x 27 = \frac{1}{2}$$	27

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5	Find the sum of $\sum_{k=7}^{\infty} 5\left(\frac{2}{5}\right)^{k-1}$ and leave in the form $\frac{a}{b}$.	$\frac{64}{1875}$

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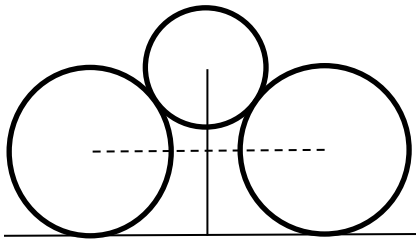
9	A small circle of radius, r = 3 cm, rests on top of two larger, identical circles of radius R = 5 cm. The two larger circles rest on a flat horizontal surface. Their centers are separated by a distance 12 cm. Determine the distance, H, from the center of the small circle to the horizontal surface. In other words, find the length of the solid vertical line shown in the diagram below. State your answer in the form $a + b\sqrt{c}$ 	$5 + 2\sqrt{7}$
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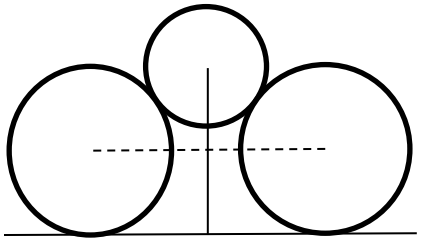
9	A small circle of radius, $r = 3$ cm, rests on top of two larger, identical circles of radius $R = 5$ cm. The two larger circles rest on a flat horizontal surface. Their centers are separated by a distance 12 cm. Determine the distance, H, from the center of the small circle to the horizontal surface. In other words, find the length of the solid vertical line shown in the diagram below. State your answer in the form $a + b\sqrt{c}$ 	$5 + 2\sqrt{7}$
10	There is a bright light source that is set at ground level and located 50 m from the foot of a house. The light source and the house are on horizontal ground. Truman is 1.6 m tall and hops in a straight line towards the house at $0.5\frac{m}{s}$. His shadow is cast upon the vertical wall of the house. How fast, in m/s, is Truman's shadow on the house shortening at the instant when he is at 30 m from the house? Express your answer as a fraction in simplest form.	$\frac{1}{10}$ Accept either + or – answers.

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2016 – 2017 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
1	1	1	$\frac{47}{2} = 23\ R.1 \rightarrow \frac{23}{2} = 11\ R.1 \rightarrow \frac{11}{2} = 5\ R.1 \rightarrow \frac{5}{2} = 2\ R.1$ $\frac{2}{2} = 1\ R.0 \rightarrow \frac{1}{2} = 0\ R.1$ 101111_2
2	2	2	$(90 - \mathrm{x}) = \text{complement}, (180 - \mathrm{x}) = \text{supplement}$ $2(180 - \mathrm{x}) = \mathrm{x} + \frac{1}{5}(90 - \mathrm{x}) + 202$ $360 - 2\mathrm{x} = \mathrm{x} + 18 - \frac{\mathrm{x}}{5} + 202$ $0 = \mathrm{x} + 18 - \frac{\mathrm{x}}{5} + 202 - 360 + 2\mathrm{x}$ $0 = \frac{14\mathrm{x}}{5} - 140 \rightarrow 140 = \frac{14\mathrm{x}}{5} \rightarrow 700 = 14\mathrm{x} \rightarrow 50 = \mathrm{x}$ Therefore, the complement of the angle is 40°
3	3	3	$\frac{1}{6} \cdot \frac{1}{7} \cdot \frac{1}{8} \cdot \frac{1}{9} \cdot \frac{1}{10}\mathrm{x} = 5 \Rightarrow \mathrm{x} = 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 = 151200$ The second digit is 5
4	4		The probability of at least one working is 1 minus the probability of its complement, which is ALL are broken. $P(X) = 1 - P'(x) = 1 - \left(\frac{1}{5}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \frac{39}{40}$ If a student interprets ALL ARE NOT BROKEN as meaning ALL ARE WORKING, and they obtain the answer $P(x) = \frac{3}{10}$, then allow this as a correct answer.
		4	$\left(\left(\sqrt[4]{\sqrt[10]{6}}\right)^2\right)^{15} = 6^{\frac{15 \cdot 2}{4 \cdot 10}} = 6^{\frac{30}{40}} = 6^{\frac{3}{4}} = \sqrt[4]{216}$

5			$f'(x) = 6x + 4k$ $f'(2) = 6(2) + 4k = 16$ $4k = 16 - 12 = 4$ $k = 1$
	5	5	$S_\infty = \frac{5}{1-\frac{2}{5}}$ $\sum_{k=7}^{\infty} 5\left(\frac{2}{5}\right)^{k-1} = 5\left(\left(\frac{2}{5}\right)^6 + \left(\frac{2}{5}\right)^7 + \cdots.\right)$ $\sum_{k=7}^{\infty} 5\left(\frac{2}{5}\right)^{k-1} = 5\left(\frac{2}{5}\right)^6\left(\left(\frac{2}{5}\right)^0 + \left(\frac{2}{5}\right)^1\right) + \cdots) = \sum_{k=0}^{\infty} 5\left(\frac{2}{5}\right)^6\left(\frac{2}{5}\right)^k$ $\frac{(5)\left(\frac{2}{5}\right)^6}{1-\frac{2}{5}} = \frac{(5^2)(2^6)}{(3)(5^6)} = \frac{64}{1875}$
6	6	6	The constant term will occur at the 7th term of the expansion. Letting $x = a$ and $y = \frac{1}{a}$, the 7th term may be expressed as $\binom{12}{6}xy = \frac{12!}{6!6!}a\left(\frac{1}{a}\right) = 924$
7	7	7	N can only be an odd number therefore N=1,3,5,7,9. $1 + 3 + 5 + 7 + 9 = 25$
8	8	8	The three best friends can sit together in 3! Ways. They as a group along with the other 6 people make 7 groups which can be permutated in 7! Ways. Therefore $3! \cdot 7! = 30240$

9	9		The length of the solid line, $H = y + R$, where y is the is the top portion of the solid line H, above the horizontal dotted line. The lower portion is equal to the radii of the larger circles. The length of the dotted line from the center of the small circle to either of the centers of the larger circles is $r + R$. According to the Pythagorean Theorem, $(r + R)^2 = y^2 + \left(\frac{x}{2}\right)^2 \rightarrow\ y^2 = (r + R)^2 - \frac{x^2}{4}$ $H = \sqrt{(r + R)^2 - \frac{x^2}{4}} + R \rightarrow\ H = \sqrt{8^2 - \frac{144}{4}} + 5$ $H = \sqrt{64 - 36} + 5 = 5 + 2\sqrt{7}$
		9	$(3^2 + (5 \cdot 4))\#(5^2 + (5 \cdot 6)) = 29\#55 = 29^2 + (5 \cdot 55) = 1116$
10			Consider a right triangle, ΔABC, the light is at A and the side BC is Truman's shadow. There is another right triangle, ΔADE, where the side DE is Truman himself. These two triangles are similar. The following proportion may be setup. Let BC equal the height of Truman's shadow and 1.6 m be the length of Truman, DE, $\frac{H}{50} = \frac{1.6}{x} \rightarrow\ H = 80x^{-1}$ $\frac{dH}{dt} = -80x^{-2}\frac{dx}{dt}$ At 30 m from the house, x = 20 m, $\frac{dH}{dt} = -80(20)^{-2}\left(\frac{1}{2}\ ms^{-1}\right) = -\frac{1}{10}\frac{m}{s}$

	10	10	The inscribed triangle can be subdivided into 3 triangles by drawing a line from the center of the circle to each vertex. These radial lines bisect each angle of the triangle. Therefore, an isosceles triangle with base 48 and base angles each 30 degrees may be analyzed. The height of this triangle is drawn to the center of the circle, making 1 30-60-90 right triangle and bisecting its base. The following trig relationship holds: $\text{Cos}\,(30) = \frac{24}{\text{r}} \rightarrow \text{r} = \frac{48}{\sqrt{3}}$ The area of the triangle is thus, $\pi\left(\frac{48^2}{3}\right) = 768\pi$
11	11	11	1st condition: A chartreuse button from the silver bag is chosen with $\frac{5}{8}$ probability AND a magenta button is chosen with $\frac{2}{9}$ probability from the gold bag after 4 chartreuse buttons are added to it. OR 2nd condition: A magenta button is chosen from the silver bag with $\frac{3}{8}$ probability AND a magenta button is chosen with $\frac{8}{11}$ probability from the gold bag after 6 magenta buttons are added to it. Therefore: $(\frac{5}{8} \cdot \frac{2}{9}) + (\frac{3}{8} \cdot \frac{8}{11}) = \frac{163}{396}$
12	12	12	The total number of possible start time combinations for the duo is $5^2 = 25$, since each of them has the choice of starting at 8,9,10,11 or 12. The times that will allow for their paths to intersect are: Truman: 8 → Colin: 8,9,10 Truman: 9 → Colin: 8,9,10,11 Truman: 10 → Colin: 8,9,10,11,12 Truman: 11 → Colin: 9,10,11,12 Truman: 12 → Colin: 10,11,12 There are 19 possible time combinations where they would meet, either on the trail or at the trail entry point. The probability is 19/25

13	13	13	Let d=number of dollars and c=number of cents, then $100c + d + 65 = \frac{1}{2}(100d + c)$ $d = \frac{199c + 130}{98} = 2c + \frac{3c + 32}{98} + 1$ Since d and c are both integers between 0 and 99, c must equal 22. Thus, $d = \frac{199*22+130}{98} = 46$
14	14		In standard form, the equation of this ellipse is written as $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$. This gives the semi-major axis in the x direction, a=3 ft, and the semi-minor axis in the y direction, b=2 ft. The area of this ellipse is $\pi ab = 6\pi$ ft^2. The volume of the tube is found by multiplying the area of this ellipse by the axial length of the tube, which is a helix that rotates one full circle while moving up. The length of this helical line is $\sqrt{\text{Height}^2 + \text{Circumference}^2} = \sqrt{4^2 + (4\pi)^2}$ Or $L = 4\sqrt{1 + \pi^2}$ Thus, $V = 24\pi\sqrt{1 + \pi^2}$ ft^3
		14	The first few terms are: 2, 8, 32, 152, and 872. Since all the factorials above 5! end in 0, all the other terms end in 2. Perfect squares cannot end in 2, so there are no perfect squares in this sequence.
15			Use substitution twice! Let $u = x^2 - 2$; $\frac{1}{2}du = xdx$ → Limits; u = <0,1> Thus; $\frac{3}{2}\int_0^1 \frac{1}{u+\sqrt{u}}\, du$ Let $v^2 = u$; $2vdv = du$ → Limits; v = <0,1> Thus; $3\int_0^1 \frac{v}{v^2+v}\, dv = 3\int_0^1 \frac{1}{v+1}\, dv = 3\ln(v + 1)\Big|_0^1$ $3(\ln(1 + 1) - \ln(0 + 1)) = 3\ln\left(\frac{2}{1}\right) = 3\ln 2 = \ln 8$

	15	15	$\frac{\log_3 x}{\log_3 729} - \frac{\log_3 27}{\log_3 x} = \frac{1}{2}$ $\frac{\log_3 x}{6} - \frac{3}{\log_3 x} = \frac{1}{2}$ $(\log_3 x)^2 - 18 = 3\log_3 x$ $(\log_3 x)^2 - 3(\log_3 x) - 18 = 0$ $(\log_3 x - 6)(\log_3 x + 3) = 0$ $\log_3 x = 6$ or $\log_3 x = -3$ $x = 3^6$ or $x = 3^{-3}$ Product of the two solutions: $3^3 = 27$