2015 – 2016 Log1 Contest Round 3
Theta Individual

Name: ________________

4 points each		
1	Find the square root of the sum of the 2016 least positive odd integers.	
2	An archaic trigonometric function known as versine is defined as $\operatorname{ver}\sin\theta = 1 - \cos\theta$. Find the value of $\operatorname{ver}\sin 60^\circ$.	
3	For Valentine's Day, Mary received several edible Valentine's Day hearts. On Valentine's Day, she ate 20% of the hearts. The next day, she ate 25% of the hearts that were remaining, leaving her with 72 hearts. How many Valentine's Day hearts did Mary initially receive?	
4	The Man from Another Place has twice as many dance moves as The Giant and BOB combined. The Giant has three more dance moves than BOB and nine fewer dance moves than The Man from Another Place. How many dance moves does The Man from Another Place have?	
5	25% of 150 is what percent of 25?	

5 points each		
6	How many real solutions does the equation $12x^2 - 17\|x\| + 6 = 0$ have?	
7	If *x* and *y* are positive real numbers, let $A(x,y)$, $G(x,y)$, and $H(x,y)$ be the arithmetic, geometric, and harmonic means of *x* and *y*, respectively. Find the numerical value of $\dfrac{G(A(2015,2016),H(2015,2016))}{G(2015,2016)}$.	
8	A regular icosahedron and a regular octahedron, both of which have edges of length 4 inches, are fused together by connecting one face of each so that the edges of those faces coincide. Find the surface area, in square inches, of this new figure.	
9	In the little-known sport of fooooootball, touchdowns are worth 13 points each, field goals are worth 8 points each, and safeties are worth 3 points each. Further, those are the only ways to score points in fooooootball. What is the maximum unattainable score in fooooootball?	
10	Find the non-vertical, non-horizontal asymptote of the function $f(x) = \dfrac{6x^2 - 3x + 2}{2x - 3}$.	

6 points each		
11	Simplify: $\sqrt{100 \cdot 103 \cdot 106 \cdot 109 + 81}$	
12	A regular octagon is formed by cutting the corners off of a square at a 45° angle relative to the sides of the square. Find the ratio of the length of a side of the octagon to the length of a side of the square.	
13	Of the 2017 numbers on the row of Pascal's Triangle whose first two numbers are 1 and 2016, how many are odd?	
14	1,180,416 and 1,181,953 are consecutive triangular numbers. Given that information, simplify: $\sqrt[3]{1,181,953^2 - 1,180,416^2}$.	
15	You enter a room and find three large trunks and a wise man. The wise man tells you that one trunk is empty, one trunk is filled with treasure, and one trunk contains a poisonous gas that will kill you instantly the moment the trunk containing it is opened. Each trunk has an inscription on it, and the wise man tells you that the inscription on the trunk containing the treasure is true while the inscription on the trunk containing the poisonous gas is false. The inscriptions on the three trunks are as follows: Trunk 1: "Trunk 3 is empty." Trunk 2: "The poisonous gas is in Trunk 1." Trunk 3: "This trunk is empty." Which trunk contains the treasure?	

2015 – 2016 Log1 Contest Round 3
Alpha Individual

Name: ________________

4 points each		
1	Find the square root of the sum of the 2016 least positive odd integers.	
2	An archaic trigonometric function known as versine is defined as $\operatorname{ver}\sin\theta = 1-\cos\theta$. For how many angles θ, $0 \le \theta < 2\pi$, is $\operatorname{ver}\sin\theta = \cos\theta$?	
3	The Man from Another Place has twice as many dance moves as The Giant and BOB combined. The Giant has three more dance moves than BOB and nine fewer dance moves than The Man from Another Place. How many dance moves does The Man from Another Place have?	
4	How many real solutions does the equation $12x^2 - 17\lvert x\rvert + 6 = 0$ have?	
5	If x and y are positive real numbers, let $A(x,y)$, $G(x,y)$, and $H(x,y)$ be the arithmetic, geometric, and harmonic means of x and y, respectively. Find the numerical value of $\dfrac{G(A(2015,2016),H(2015,2016))}{G(2015,2016)}$.	

5 points each		
6	Find the distance between the foci of the conic section with equation $3x^2 + 2y^2 - 12x + 12y + 18 = 0$.	
7	How many ordered triples (a,b,c) of integers, where $0 < a \le b \le c$, satisfy the equation $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}=1$?	
8	In the little-known sport of fooooootball, touchdowns are worth 13 points each, field goals are worth 8 points each, and safeties are worth 3 points each. Further, those are the only ways to score points in fooooootball. What is the maximum unattainable score in fooooootball?	
9	A regular hexagon encloses an area of $294\sqrt{3}$ square meters. Find the sum of the lengths, in meters, of the diagonals of this hexagon.	
10	Simplify: $\sqrt{1000\cdot 1003\cdot 1006\cdot 1009 + 81}$	

6 points each		
11	A regular octagon is formed by cutting the corners off of a square at a 45° angle relative to the sides of the square. Find the ratio of the length of a side of the octagon to the length of a side of the square.	
12	Evaluate: $\begin{vmatrix} 4 & 0 & 1 & 4 \\ 0 & 0 & 2 & 2 \\ 7 & 3 & 1 & 1 \\ 2 & 0 & 3 & 7 \end{vmatrix}$	
13	Of the 2017 numbers on the row of Pascal's Triangle whose first two numbers are 1 and 2016, how many are odd?	
14	Evaluate: $\sum_{n=1}^{2016}\left(\left(-1\right)^n \cdot n^2\right)$	
15	You enter a room and find three large trunks and a wise man. The wise man tells you that one trunk is empty, one trunk is filled with treasure, and one trunk contains a poisonous gas that will kill you instantly the moment the trunk containing it is opened. Each trunk has an inscription on it, and the wise man tells you that the inscription on the trunk containing the treasure is true while the inscription on the trunk containing the poisonous gas is false. The inscriptions on the three trunks are as follows: Trunk 1: "Trunk 3 is empty." Trunk 2: "The poisonous gas is in Trunk 1." Trunk 3: "This trunk is empty." Which trunk contains the treasure?	

2015 – 2016 Log1 Contest Round 3
Mu Individual

Name: ________________

4 points each		
1	Find the square root of the sum of the 2016 least positive odd integers.	
2	An archaic trigonometric function known as versine is defined as $\operatorname{ver}\sin\theta = 1-\cos\theta$. Find the sum of the angles θ, $0 \le \theta < 2\pi$, for which $\operatorname{ver}\sin\theta = \sin\theta$.	
3	If $f(2)=4, g(2)=-5, f'(2)=3$, and $g'(2)=6$, find the value of $\left.\frac{d(f(x)g(x))}{dx}\right\vert_{x=2}$.	
4	How many real solutions does the equation $12x^2-17\lvert x\rvert+6=0$ have?	
5	If *x* and *y* are positive real numbers, let $A(x,y)$, $G(x,y)$, and $H(x,y)$ be the arithmetic, geometric, and harmonic means of *x* and *y*, respectively. Find the numerical value of $\frac{G(A(2015,2016),H(2015,2016))}{G(2015,2016)}$.	

5 points each		
6	How many ordered triples (a,b,c) of integers, where $0<a\le b\le c$, satisfy the equation $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$?	
7	Find the distance between the points that are relative extremes of the function $f(x)=x^3-3x^2-24x$.	
8	In the little-known sport of foooooootball, touchdowns are worth 13 points each, field goals are worth 8 points each, and safeties are worth 3 points each. Further, those are the only ways to score points in foooooootball. What is the maximum unattainable score in foooooootball?	
9	Simplify: $\sqrt{2016\cdot 2019\cdot 2022\cdot 2025+81}$	
10	Find the area enclosed by the graphs of $y=\sqrt{16-x^2}$ and $y=f(x)$, where $f(x)=$ the maximum number in the set $\{x,0\}$.	

6 points each		
11	A regular octagon is formed by cutting the corners off of a square at a 45° angle relative to the sides of the square. Find the ratio of the length of a side of the octagon to the length of a side of the square.	
12	Evaluate: $\int_0^{\pi/4}\left(\cos^2 x+\sin^2 x+\tan^2 x\right)dx$	
13	Of the 2017 numbers on the row of Pascal's Triangle whose first two numbers are 1 and 2016, how many are odd?	
14	Find the volume generated when the region bounded by the graphs of $y=x^2$ and $y=\sqrt{x}$ is revolved about the line $x=1$.	
15	You enter a room and find three large trunks and a wise man. The wise man tells you that one trunk is empty, one trunk is filled with treasure, and one trunk contains a poisonous gas that will kill you instantly the moment the trunk containing it is opened. Each trunk has an inscription on it, and the wise man tells you that the inscription on the trunk containing the treasure is true while the inscription on the trunk containing the poisonous gas is false. The inscriptions on the three trunks are as follows: Trunk 1: "Trunk 3 is empty." Trunk 2: "The poisonous gas is in Trunk 1." Trunk 3: "This trunk is empty." Which trunk contains the treasure?	

2015 – 2016 Log1 Contest Round 3
Theta Individual

Name: ________________

4 points each		
1	Find the square root of the sum of the 2016 least positive odd integers.	2016
2	An archaic trigonometric function known as versine is defined as $\operatorname{ver}\sin\theta = 1-\cos\theta$. Find the value of $\operatorname{ver}\sin 60^\circ$.	$\frac{1}{2}$ or 0.5
3	For Valentine's Day, Mary received several edible Valentine's Day hearts. On Valentine's Day, she ate 20% of the hearts. The next day, she ate 25% of the hearts that were remaining, leaving her with 72 hearts. How many Valentine's Day hearts did Mary initially receive?	120
4	The Man from Another Place has twice as many dance moves as The Giant and BOB combined. The Giant has three more dance moves than BOB and nine fewer dance moves than The Man from Another Place. How many dance moves does The Man from Another Place have?	14
5	25% of 150 is what percent of 25?	150 or 150%

5 points each		
6	How many real solutions does the equation $12x^2 - 17\lvert x\rvert + 6 = 0$ have?	4
7	If x and y are positive real numbers, let $A(x,y)$, $G(x,y)$, and $H(x,y)$ be the arithmetic, geometric, and harmonic means of x and y, respectively. Find the numerical value of $\frac{G(A(2015,2016),H(2015,2016))}{G(2015,2016)}$.	1
8	A regular icosahedron and a regular octahedron, both of which have edges of length 4 inches, are fused together by connecting one face of each so that the edges of those faces coincide. Find the surface area, in square inches, of this new figure.	$104\sqrt{3}$
9	In the little-known sport of foooooootball, touchdowns are worth 13 points each, field goals are worth 8 points each, and safeties are worth 3 points each. Further, those are the only ways to score points in foooooootball. What is the maximum unattainable score in foooooootball?	10
10	Find the non-vertical, non-horizontal asymptote of the function $f(x) = \frac{6x^2 - 3x + 2}{2x - 3}$.	$y = 3x + 3$

6 points each		
11	Simplify: $\sqrt{100\cdot 103\cdot 106\cdot 109+81}$	10,909
12	A regular octagon is formed by cutting the corners off of a square at a 45° angle relative to the sides of the square. Find the ratio of the length of a side of the octagon to the length of a side of the square.	$\sqrt{2}-1$
13	Of the 2017 numbers on the row of Pascal's Triangle whose first two numbers are 1 and 2016, how many are odd?	64
14	1,180,416 and 1,181,953 are consecutive triangular numbers. Given that information, simplify: $\sqrt[3]{1{,}181{,}953^2-1{,}180{,}416^2}$.	1537
15	You enter a room and find three large trunks and a wise man. The wise man tells you that one trunk is empty, one trunk is filled with treasure, and one trunk contains a poisonous gas that will kill you instantly the moment the trunk containing it is opened. Each trunk has an inscription on it, and the wise man tells you that the inscription on the trunk containing the treasure is true while the inscription on the trunk containing the poisonous gas is false. The inscriptions on the three trunks are as follows: Trunk 1: "Trunk 3 is empty." Trunk 2: "The poisonous gas is in Trunk 1." Trunk 3: "This trunk is empty." Which trunk contains the treasure?	Trunk 1

2015 – 2016 Log1 Contest Round 3
Alpha Individual

Name: ________________

	4 points each	
1	Find the square root of the sum of the 2016 least positive odd integers.	2016
2	An archaic trigonometric function known as versine is defined as $\operatorname{ver}\sin\theta = 1 - \cos\theta$. For how many angles θ, $0 \le \theta < 2\pi$, is $\operatorname{ver}\sin\theta = \cos\theta$?	2
3	The Man from Another Place has twice as many dance moves as The Giant and BOB combined. The Giant has three more dance moves than BOB and nine fewer dance moves than The Man from Another Place. How many dance moves does The Man from Another Place have?	14
4	How many real solutions does the equation $12x^2 - 17\lvert x\rvert + 6 = 0$ have?	4
5	If x and y are positive real numbers, let $A(x,y)$, $G(x,y)$, and $H(x,y)$ be the arithmetic, geometric, and harmonic means of x and y, respectively. Find the numerical value of $\dfrac{G(A(2015,2016),H(2015,2016))}{G(2015,2016)}$.	1

	5 points each	
6	Find the distance between the foci of the conic section with equation $3x^2 + 2y^2 - 12x + 12y + 18 = 0$.	$2\sqrt{2}$
7	How many ordered triples (a,b,c) of integers, where $0 < a \le b \le c$, satisfy the equation $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$?	3
8	In the little-known sport of fooooootball, touchdowns are worth 13 points each, field goals are worth 8 points each, and safeties are worth 3 points each. Further, those are the only ways to score points in fooooootball. What is the maximum unattainable score in fooooootball?	10
9	A regular hexagon encloses an area of $294\sqrt{3}$ square meters. Find the sum of the lengths, in meters, of the diagonals of this hexagon.	$84 + 84\sqrt{3}$
10	Simplify: $\sqrt{1000 \cdot 1003 \cdot 1006 \cdot 1009 + 81}$	1,009,009

6 points each		
11	A regular octagon is formed by cutting the corners off of a square at a 45° angle relative to the sides of the square. Find the ratio of the length of a side of the octagon to the length of a side of the square.	$\sqrt{2}-1$
12	Evaluate: $\begin{vmatrix} 4 & 0 & 1 & 4 \\ 0 & 0 & 2 & 2 \\ 7 & 3 & 1 & 1 \\ 2 & 0 & 3 & 7 \end{vmatrix}$	-60
13	Of the 2017 numbers on the row of Pascal's Triangle whose first two numbers are 1 and 2016, how many are odd?	64
14	Evaluate: $\sum_{n=1}^{2016}\left((-1)^n \cdot n^2\right)$	2,033,136
15	You enter a room and find three large trunks and a wise man. The wise man tells you that one trunk is empty, one trunk is filled with treasure, and one trunk contains a poisonous gas that will kill you instantly the moment the trunk containing it is opened. Each trunk has an inscription on it, and the wise man tells you that the inscription on the trunk containing the treasure is true while the inscription on the trunk containing the poisonous gas is false. The inscriptions on the three trunks are as follows: Trunk 1: "Trunk 3 is empty." Trunk 2: "The poisonous gas is in Trunk 1." Trunk 3: "This trunk is empty." Which trunk contains the treasure?	Trunk 1

2015 – 2016 Log1 Contest Round 3
Mu Individual

Name: ________________

	4 points each	
1	Find the square root of the sum of the 2016 least positive odd integers.	2016
2	An archaic trigonometric function known as versine is defined as $\operatorname{ver}\sin\theta = 1-\cos\theta$. Find the sum of the angles $\theta,\ 0\le\theta<2\pi$, for which $\operatorname{ver}\sin\theta=\sin\theta$.	$\frac{\pi}{2}$
3	If $f(2)=4, g(2)=-5, f'(2)=3,$ and $g'(2)=6$, find the value of $\left.\frac{d\left(f(x)g(x)\right)}{dx}\right\rvert_{x=2}$.	9
4	How many real solutions does the equation $12x^2-17\lvert x\rvert+6=0$ have?	4
5	If *x* and *y* are positive real numbers, let $A(x,y)$, $G(x,y)$, and $H(x,y)$ be the arithmetic, geometric, and harmonic means of *x* and *y*, respectively. Find the numerical value of $\frac{G\left(A(2015,2016),H(2015,2016)\right)}{G(2015,2016)}$.	1

	5 points each	
6	How many ordered triples (a,b,c) of integers, where $0<a\le b\le c$, satisfy the equation $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$?	3
7	Find the distance between the points that are relative extremes of the function $f(x)=x^3-3x^2-24x$.	$30\sqrt{13}$
8	In the little-known sport of fooooootball, touchdowns are worth 13 points each, field goals are worth 8 points each, and safeties are worth 3 points each. Further, those are the only ways to score points in fooooootball. What is the maximum unattainable score in fooooootball?	10
9	Simplify: $\sqrt{2016\cdot2019\cdot2022\cdot2025+81}$	4,082,409
10	Find the area enclosed by the graphs of $y=\sqrt{16-x^2}$ and $y=f(x)$, where $f(x)=$ the maximum number in the set $\{x,0\}$.	6π

	6 points each	
11	A regular octagon is formed by cutting the corners off of a square at a 45° angle relative to the sides of the square. Find the ratio of the length of a side of the octagon to the length of a side of the square.	$\sqrt{2}-1$
12	Evaluate: $\int_0^{\pi/4}\left(\cos^2 x+\sin^2 x+\tan^2 x\right)dx$	1
13	Of the 2017 numbers on the row of Pascal's Triangle whose first two numbers are 1 and 2016, how many are odd?	64
14	Find the volume generated when the region bounded by the graphs of $y=x^2$ and $y=\sqrt{x}$ is revolved about the line $x=1$.	$\frac{11\pi}{30}$
15	You enter a room and find three large trunks and a wise man. The wise man tells you that one trunk is empty, one trunk is filled with treasure, and one trunk contains a poisonous gas that will kill you instantly the moment the trunk containing it is opened. Each trunk has an inscription on it, and the wise man tells you that the inscription on the trunk containing the treasure is true while the inscription on the trunk containing the poisonous gas is false. The inscriptions on the three trunks are as follows: Trunk 1: "Trunk 3 is empty." Trunk 2: "The poisonous gas is in Trunk 1." Trunk 3: "This trunk is empty." Which trunk contains the treasure?	Trunk 1

2015 – 2016 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
1	1	1	$\sqrt{\sum_{n=1}^{2016}(2n-1)}=\sqrt{2016^2}=2016$
		2	$\operatorname{ver}\sin 60^\circ=1-\cos 60^\circ=1-\frac{1}{2}=\frac{1}{2}$
	2		$\cos\theta=\operatorname{ver}\sin\theta=1-\cos\theta\Rightarrow 2\cos\theta=1\Rightarrow\cos\theta=\frac{1}{2}$, and cosine has positive values in quadrants II and IV, so there are two such solutions (they are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$).
2			$\sin\theta=\operatorname{ver}\sin\theta=1-\cos\theta\Rightarrow 1=\cos\theta+\sin\theta$. Squaring both sides of this equation, $1=\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta=1+2\sin\theta\cos\theta\Rightarrow\sin\theta=0$ or $\cos\theta=0$. However, each of these solutions can only be paired with the other trigonometric function equaling 1, so the solutions are $\sin\theta=0,\ \cos\theta=1\Rightarrow\theta=0$ and $\cos\theta=0,\ \sin\theta=1\Rightarrow\theta=\frac{\pi}{2}$. The sum of these solutions is $0+\frac{\pi}{2}=\frac{\pi}{2}$.
		3	Let x be the number of hearts Mary initially had. After Valentine's Day, Mary had $\frac{4}{5}$ of her hearts remaining, or $\frac{4}{5}x$. The next day, after eating, Mary had $\frac{3}{4}$ of that remaining, or $\frac{3}{4}\cdot\frac{4}{5}x=\frac{3}{5}x$. Therefore, $\frac{3}{5}x=72\Rightarrow x=120$.
	3	4	Let M, G, and B be the number of dance moves of The Man from Another Place, The Giant, and BOB, respectively. Then $M=2(G+B)$, $G-3=B$, and $M=G+9$. Substituting the second two equations into the first one, $G+9=2(G+G-3)\Rightarrow G=5$. Therefore, The Man from Another Place has $5+9=14$ dance moves.
3			$\left.\frac{d\left(f(x)g(x)\right)}{dx}\right\|_{x=2}=f(2)g'(2)+g(2)f'(2)=4\cdot 6+(-5)\cdot 3=24-15=9$
		5	25% of 150 is 37.5, and this is 150% of 25.
4	4	6	$0=12x^2-17\lvert x\rvert+6=(3\lvert x\rvert-2)(4\lvert x\rvert-3)\Rightarrow\lvert x\rvert=\frac{2}{3}$ or $\lvert x\rvert=\frac{3}{4}$. Therefore, there are four solutions to this equation.
5	5	7	$A(x,y)=\frac{x+y}{2}$, $G(x,y)=\sqrt{xy}$, and $H(x,y)=\frac{2xy}{x+y}$. Therefore, $G\left(A(x,y),H(x,y)\right)$ $=G\left(\frac{x+y}{2},\frac{2xy}{x+y}\right)=\sqrt{\left(\frac{x+y}{2}\right)\left(\frac{2xy}{x+y}\right)}=\sqrt{xy}=G(x,y)$, so $\frac{G\left(A(2015,2016),H(2015,2016)\right)}{G(2015,2016)}=\frac{G(2015,2016)}{G(2015,2016)}=1$

	6		$0=3x^2+2y^2-12x+12y+18 \Rightarrow 12=3(x-2)^2+2(y+3)^2 \Rightarrow 1=\frac{(x-2)^2}{4}+\frac{(y+3)^2}{6}$, so the distance from the center of one focus is $\sqrt{6-4}=\sqrt{2}$, making the distance between the foci $2\sqrt{2}$.
6	7		*a* cannot equal 1 as the sum would be too large, and *a* cannot equal 4 or more since, having the greatest reciprocal, the sum would be too small; therefore, $a=2$ or $a=3$. Further, the only solution with $a=3$ is $(3,3,3)$, as increasing *b* or *c* would make the sum too small. Therefore, we turn our attention to $a=2$, which reduces the equation to $\frac{1}{b}+\frac{1}{c}=\frac{1}{2}$, where $2 \le b \le c$. b cannot equal 2 nor equal 5 or more by similar reasoning as above, so $b=3$ or $b=4$, and $b=4$ has only the solution $(2,4,4)$, also by similar reasoning. Therefore, we let $b=3$, which yields only $c=6$. Therefore, there are three solutions (which are $(3,3,3)$, $(2,4,4)$, and $(2,3,6)$).
7			Since *f* is a cubic function the relative extremes occur when $f'(x)=3x^2-6x-24$ equals 0. $0=3x^2-6x-24=3(x-4)(x+2) \Rightarrow x=4$ or $x=-2$. $f(4)=-80$ and $f(-2)=28$, so the distance between the points $(4,-80)$ and $(-2,28)$ is $\sqrt{(4+2)^2+(-80-28)^2}=\sqrt{11700}=30\sqrt{13}$.
		8	Both solids have equilateral triangle faces of side length 4 inches, so each face has area $\frac{4^2\sqrt{3}}{4}=4\sqrt{3}$ inches. The icosahedron has 20 faces and the octahedron has 8 faces, but the fusing of two faces, one from each, means that neither of those faces will be on the surface anymore. Therefore, there are 26 faces, and the total surface area of the figure is $26 \cdot 4\sqrt{3}=104\sqrt{3}$ square inches.
8	8	9	Break the positive integers into three groups: those divisible by 3, those that leave a remainder of 1 when divided by 3, and those that leave a remainder of 2 when divided by 3. Obviously all scores in the group of numbers divisible by 3 can be achieved by scoring however many safeties (3 points each). In the group of numbers that leave remainder 1, 13 can be achieved by scoring a touchdown, and any score greater than 13 in this group can be achieved by scoring however many safeties (since each score greater is some multiple of 3 greater than 13); 10 cannot be achieved however, since no whole number of 3s and/or 8s sum to 10 (which also means no score below 10 in this group can be achieved either). In the group of numbers that leave remainder 2, 8 can be achieved by scoring a field goal, and any score greater than 8 in this group can be achieved by scoring however many safeties (since each score greater is some multiple of 3 greater than 8); 5 cannot be achieved however, since no whole numbers of 3s sum to 5 (which also means no score below 5 in this group can be achieved either). Therefore, looking at all three cases, 10 is the maximum score that cannot be achieved.

	9		The length s of a side of the hexagon satisfies $\frac{3s^2\sqrt{3}}{2}=294\sqrt{3}\Rightarrow s=14$ meters. The hexagon has $\frac{(6)(6-3)}{2}=9$ total diagonals: 3 long diagonals (connecting vertices opposite the hexagon from each other) and 6 short diagonals (connecting vertices that have exactly one vertex between them). The long diagonals are twice the length of a side of the hexagon, so that total length is $3\cdot(2\cdot 14)=84$ meters, and the short diagonals are $\sqrt{3}$ times as long as a side, so that total length is $6\cdot\left(\sqrt{3}\cdot 14\right)=84\sqrt{3}$ meters (these two relationships can be found using simple right triangle trigonometry). Thus the total length of all diagonals is $84+84\sqrt{3}$ meters.
9			Let $x=2016$. Then $\sqrt{2016\cdot 2019\cdot 2022\cdot 2025+81}=\sqrt{x(x+3)(x+6)(x+9)+81}$ $=\sqrt{(x^2+9x)(x^2+9x+18)+81}=\sqrt{(x^2+9x)^2+18(x^2+9x)+81}=\sqrt{(x^2+9x+9)^2}$ $=x^2+9x+9=2016^2+9(2016)+9=4{,}082{,}409$.
		10	Using long division, $\frac{6x^2-3x+2}{2x-3}=3x+3+\frac{11}{2x-3}$, so the asymptote is $y=3x+3$.
	10		Let $x=1000$. Then $\sqrt{1000\cdot 1003\cdot 1006\cdot 1009+81}=\sqrt{x(x+3)(x+6)(x+9)+81}$ $=\sqrt{(x^2+9x)(x^2+9x+18)+81}=\sqrt{(x^2+9x)^2+18(x^2+9x)+81}=\sqrt{(x^2+9x+9)^2}$ $=x^2+9x+9=1000^2+9(1000)+9=1{,}009{,}009$.
10			Another way to write f is $f(x)=\begin{cases}x, \text{ if } x\geq 0\\ 0, \text{ if } x<0\end{cases}$, which is two rays emanating from the origin: one along the negative x-axis, the other along $y=x$. The other graph is the upper-half semicircle centered at the origin with radius 4. Therefore, the region is the entire second quadrant of the enclosed circle ($A=\frac{1}{4}(\pi\cdot 4^2)=4\pi$) and half of the first quadrant of the enclosed circle ($A=\frac{1}{2}\cdot\frac{1}{4}(\pi\cdot 4^2)=2\pi$). Therefore, the total area enclosed by these graphs is $4\pi+2\pi=6\pi$.
		11	Let $x=100$. Then $\sqrt{100\cdot 103\cdot 106\cdot 109+81}=\sqrt{x(x+3)(x+6)(x+9)+81}$ $=\sqrt{(x^2+9x)(x^2+9x+18)+81}=\sqrt{(x^2+9x)^2+18(x^2+9x)+81}=\sqrt{(x^2+9x+9)^2}$ $=x^2+9x+9=100^2+9(100)+9=10{,}909$.
11	11	12	Let x be the side length of the octagon. Since the removed corners are $45^\circ-45^\circ-90^\circ$ right triangles, the legs are each of length $\frac{x}{\sqrt{2}}=\frac{x\sqrt{2}}{2}$. Each side of the square is made up of one octagon side length and two right triangle legs, so the side length of the square is $x+2\left(\frac{x\sqrt{2}}{2}\right)=x\left(1+\sqrt{2}\right)$. Therefore, the ratio of a side length of the octagon to that of the square is $\frac{x}{x\left(1+\sqrt{2}\right)}=\sqrt{2}-1$.

	12		Expanding by the second column, $\begin{vmatrix} 4 & 0 & 1 & 4 \\ 0 & 0 & 2 & 2 \\ 7 & 3 & 1 & 1 \\ 2 & 0 & 3 & 7 \end{vmatrix} = -3\begin{vmatrix} 4 & 1 & 4 \\ 0 & 2 & 2 \\ 2 & 3 & 7 \end{vmatrix}$. Now, using the diagonal rule, $-3\begin{vmatrix} 4 & 1 & 4 \\ 0 & 2 & 2 \\ 2 & 3 & 7 \end{vmatrix} = -3(56+4+0-16-24-0) = -3(20) = -60$.
12			$\int_0^{\pi/4}(\cos^2 x + \sin^2 x + \tan^2 x)dx = \int_0^{\pi/4}(1+\tan^2 x)dx = \int_0^{\pi/4}(\sec^2 x)dx = \tan x\Big\vert_0^{\pi/4} = \tan\frac{\pi}{4}$ $-\tan 0 = 1 - 0 = 1$
13	13	13	The entries in that row of Pascal's Triangle will be the coefficients in the expansion of $(1+x)^{2016}$. Using the fact that $\binom{2^n}{k}$ is even for integers $1 \le k \le 2^n - 1$ (which can be proven easily by induction) and that every positive integer can be written uniquely as a sum of powers of 2 (consider the number written in binary to see that this is true), we know that $2016 = 1024+512+256+128+64+32$, so $(1+x)^{2016} = (1+x)^{1024}$ $(1+x)^{512}(1+x)^{256}(1+x)^{128}(1+x)^{64}(1+x)^{32}$, and since every term in this expansion is one term out of each smaller expansion, and the only odd terms in each smaller expansion are 1 and x^n for $n=1024$, 512, 256, 128, 64, and 32, there are two choices of term in each smaller expansion, and 6 smaller expansions total, meaning there are $2^6 = 64$ odd terms in this row (as it turns out, the number of odd entries in the row of Pascal's Triangle with terms of the form $\binom{n}{k}$ is 2 raised to the number of 1s in the binary representation of *n*).
		14	First, note that $1{,}181{,}953 - 1{,}180{,}416 = 1537$, so $1{,}180{,}416 = 1+2+\ldots+1536$ $= \frac{1536 \cdot 1537}{2}$ and $1{,}181{,}953 = 1+2+\ldots+1537 = \frac{1537 \cdot 1538}{2}$. Therefore, $\sqrt[3]{1{,}181{,}953^2 - 1{,}180{,}416^2} = \sqrt[3]{\left(\frac{1537 \cdot 1538}{2}\right)^2 - \left(\frac{1536 \cdot 1537}{2}\right)^2}$ $= \sqrt[3]{\frac{1537^2}{4}(1538^2 - 1536^2)} = \sqrt[3]{\frac{1537^2}{4}(1538-1536)(1538+1536)} = \sqrt[3]{1537^3} = 1537$.
	14		Combining the terms pairwise, $\sum_{n=1}^{2016}\left((-1)^n \cdot n^2\right) = (2^2 - 1^2) + (4^2 - 3^2) + (6^2 - 5^2) + \ldots$ $+(2016^2 - 2015^2) = 3 + 7 + 11 + \ldots + 4031$, which is arithmetic with 1008 terms. Therefore, the sum is $\frac{1008}{2}(3+4031) = 2{,}033{,}136$.

14			The graphs intersect when $x=0$ and $x=1$, and using the shells method, the radius of a shell is $1-x$ while the height of a shell is $\sqrt{x}-x^2$, so the volume is $2\pi\int_0^1(1-x)\left(\sqrt{x}-x^2\right)dx=2\pi\int_0^1\left(\sqrt{x}-x\sqrt{x}-x^2+x^3\right)dx=2\pi\left(\frac{2}{3}x^{3/2}-\frac{2}{5}x^{5/2}-\frac{1}{3}x^3+\frac{1}{4}x^4\right)\Big\|_0^1$ $=2\pi\left(\frac{2}{3}-\frac{2}{5}-\frac{1}{3}+\frac{1}{4}\right)=2\pi\left(\frac{11}{60}\right)=\frac{11\pi}{30}$.
15	15	15	The treasure could not be in Trunk 3, since if it was, then the inscription on Trunk 3, "This trunk is empty.", must be true, a contradiction. The treasure could not be in Trunk 2 either, since if it was, then the inscription on Trunk 2, "The poisonous gas is in Trunk 1.", must be true, meaning that the inscription on Trunk 1, "Trunk 3 is empty.", must be false. Since in this case Trunk 2 contains the treasure and Trunk 1 contains the poisonous gas, Trunk 3 must be empty, another contradiction. Therefore, the treasure must be in Trunk 1 (further implying that the poisonous gas is in Trunk 2 and Trunk 3 is empty).