2015 – 2016 Log1 Contest Round 2
Theta Sequences & Series

Name: ________________

	4 points each	
1	Find the 2016th term of the sequence $1,\ 2,\ ...,\ n,\ ...$.	
2	How many triangles exist whose sides have lengths equal to three consecutive numbers from the Fibonacci sequence?	
3	Consider the sequence $1, 2, 3, 1, 2, 3, ..., 1, 2, 3, ...$. Find the 2016th term of this sequence.	
4	Consider the sequence defined recursively by $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. If $a_1 = \ln 2$ and $a_2 = \ln 3$, find the value of a_7.	
5	An arithmetic sequence has first term 1 and common difference 2. Find the 50th term of this sequence.	

	5 points each	
6	An arithmetic sequence has first term 1 and common difference 2. Find the sum of the first 50 terms of this sequence.	
7	Evaluate: $\sum_{n=1}^{25}(20n+4)$	
8	In my house of cards, starting from the top, the first row consists of 3 cards, and each other row consists of 2 times as many cards as the row above it. If my house of cards consists of 10 rows, then it consists of how many total cards?	
9	Suppose that a sequence $\{a_n\}$ is defined by $a_n = an^2 + bn + c$, where *a*, *b*, and *c* are real numbers, and suppose that the first three terms of this sequence are $a_1 = 2, a_2 = 5$, and $a_3 = 11$. Find the fourth term a_4 of this sequence.	
10	Let *i* be the imaginary unit. Find the 2015th term of the sequence $\{i^n\}_{n=1}^{\infty}$, written in $a + bi$ form.	

6 points each		
11	Let i be the imaginary unit, and let $a_n = i^n$, where n is a natural number. Define a new sequence $\{s_n\}_{n=1}^{\infty}$ by $s_n = \sum_{i=1}^{n} a_i$. Find the value of s_{2015}.	
12	For a sequence $\{a_i\}_{i=1}^{\infty}$, the sum of the first n terms of this sequence is equal to $n^2 + 2n$. Find the numerical value of a_1.	
13	A radioactive substance initially weighs 1224 grams and has a half-life of 1 day. What is the weight, in grams, of this substance exactly 3 days after it was initially weighed?	
14	Evaluate: $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 3}{3^n}$	
15	A sequence of increasing integers is defined in the following way: any term is the geometric mean of the term before it and the term after it. If the first term of this sequence is 3, find the least possible value of the fourth term of this sequence.	

2015 – 2016 Log1 Contest Round 2
Alpha Sequences & Series

Name: ________________

4 points each		
1	Find the 2016th term of the sequence $1, 2, ..., n, ...$.	
2	How many triangles exist whose sides have lengths equal to three consecutive numbers from the Fibonacci sequence?	
3	Consider the sequence defined recursively by $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. If $a_1 = \ln 2$ and $a_2 = \ln 3$, find the value of a_7.	
4	Consider the sequence whose nth term is given by $a_n = \cos\left(\frac{n\pi}{6}\right) + i\sin\left(\frac{n\pi}{6}\right)$, where i is the imaginary unit. Find a_{2015}, written in $a + bi$ form, where a and b are real numbers.	
5	An arithmetic sequence has first term 1 and common difference 2. Find the sum of the first 50 terms of this sequence.	

5 points each		
6	In my house of cards, starting from the top, the first row consists of 2 cards, and each other row consists of 3 times as many cards as the row above it. If my house of cards consists of 10 rows, then it consists of how many total cards?	
7	Evaluate: $\sum_{n=1}^{\infty} \frac{2}{n^2 + 2n}$	
8	Suppose that a sequence $\{a_n\}$ is defined by $a_n = an^2 + bn + c$, where a, b, and c are real numbers, and suppose that the first three terms of this sequence are $a_1 = 2, a_2 = 5$, and $a_3 = 11$. Find the fourth term a_4 of this sequence.	
9	Let i be the imaginary unit, and let $a_n = i^n$, where n is a natural number. Define a new sequence $\{s_n\}_{n=1}^{\infty}$ by $s_n = \sum_{i=1}^{n} a_i$. Find the value of s_{2015}.	
10	For the sequence whose nth term is $a_n = \binom{n}{3}$, how many of the first 100 terms are divisible by 3?	

6 points each		
11	For a sequence $\{a_i\}_{i=1}^{\infty}$, the sum of the first n terms of this sequence is equal to $\frac{4}{n^2+2n}$. Find the numerical value of a_2.	
12	A radioactive substance initially weighs 1224 grams and has a half-life of 1 day. What is the weight, in grams, of this substance exactly 3 days after it was initially weighed?	
13	Let $f(x)=3x^4-12x^3+14x^2+15x-10$, and let $g_1(x)=x-1$. When f is divided by g_1 according to the Division Algorithm, the quotient is $q_1(x)$ and the remainder is real number r_1. For integer $n\geq 2$, let r_n be the remainder provided by the Division Algorithm when $q_{n-1}(x)$ is divided by $g_n(x)=x-n$. Find the numerical value of r_3.	
14	A sequence of increasing integers is defined in the following way: any term is the geometric mean of the term before it and the term after it. If the first term of this sequence is 3, find the least possible value of the fourth term of this sequence.	
15	Let $F_1=F_2=1$, and for integer $n\geq 3$, $F_n=F_{n-1}+F_{n-2}$. As n grows without bound, the ratio $\frac{F_n}{F_{n-1}}$ tends toward what number?	

2015 – 2016 Log1 Contest Round 2
Mu Sequences & Series

Name: ________________

4 points each		
1	Find the 2016th term of the sequence $1, 2, ..., n, ...$.	
2	How many triangles exist whose sides have lengths equal to three consecutive numbers from the Fibonacci sequence?	
3	Evaluate: $\sum_{n=1}^{\infty} \frac{n+3}{2^n}$	
4	Consider the sequence whose *n*th term is given by $a_n = \cos\left(\frac{n\pi}{6}\right) + i\sin\left(\frac{n\pi}{6}\right)$, where i is the imaginary unit. Find a_{2015}, written in $a+bi$ form, where *a* and *b* are real numbers.	
5	Consider the sequence whose *n*th term is given by $a_n = \int_0^{\frac{n\pi}{6}} \sin x \, dx$. Find the value of a_8.	

5 points each		
6	In my house of cards, starting from the top, the first row consists of 2 cards, and each other row consists of 3 times as many cards as the row above it. If my house of cards consists of 10 rows, then it consists of how many total cards?	
7	Evaluate: $\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$	
8	Suppose that a sequence $\{a_n\}$ is such that a_n is a polynomial expression in *n* with real coefficients. Further, suppose that the first four terms of this sequence are $a_1 = 2, a_2 = 7, a_3 = 14$, and $a_4 = 25$, and the degree of a_n is as small as possible. Find the fifth term a_5 of this sequence.	
9	Let *i* be the imaginary unit, and let $a_n = i^n$, where *n* is a natural number. Let $s_n = \sum_{i=1}^{n} a_i$ and define a new sequence $\{b_n\}_{n=1}^{\infty}$ by $b_n = \sum_{i=1}^{n} s_i$. Find the value of b_{2015}, written in $a+bi$ form.	
10	For a sequence $\{a_i\}_{i=1}^{\infty}$, the sum of the first *n* terms of this sequence is equal to $\frac{4}{n^2+2n}$. Find the numerical value of a_2.	

6 points each		
11	Let $f(x)=15x^5-14x^4+9x^2-22x+13$, and define a sequence $\{a_n\}_{n=1}^{\infty}$ by $a_n=f^{(n)}(2)$. Find the numerical value of a_4.	
12	A radioactive substance initially weighs 1224 grams and has a half-life of 1 day. What is the weight, in grams, of this substance exactly 3 days after it was initially weighed?	
13	Let $f(x)=3x^4-12x^3+14x^2+15x-10$, and let $g_1(x)=x-1$. When f is divided by g_1 according to the Division Algorithm, the quotient is $q_1(x)$ and the remainder is real number r_1. For integer $n\geq 2$, let r_n be the remainder provided by the Division Algorithm when $q_{n-1}(x)$ is divided by $g_n(x)=x-n$. Find the numerical value of r_3.	
14	A sequence of increasing integers is defined in the following way: any term is the geometric mean of the term before it and the term after it. If the first term of this sequence is 3, find the least possible value of the fourth term of this sequence.	
15	Let $F_1=F_2=1$, and for integer $n\geq 3$, $F_n=F_{n-1}+F_{n-2}$. Find the value of $\lim_{n\to\infty}\frac{F_n}{F_{n-1}}$.	

2015 – 2016 Log1 Contest Round 2
Theta Sequences & Series

Name: ________________

	4 points each	
1	Find the 2016th term of the sequence $1,\ 2,\ ...,\ n,\ ...$.	2016
2	How many triangles exist whose sides have lengths equal to three consecutive numbers from the Fibonacci sequence?	0
3	Consider the sequence $1, 2, 3, 1, 2, 3, ..., 1, 2, 3, ...$. Find the 2016th term of this sequence.	3
4	Consider the sequence defined recursively by $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. If $a_1 = \ln 2$ and $a_2 = \ln 3$, find the value of a_7.	$\ln 209952$
5	An arithmetic sequence has first term 1 and common difference 2. Find the 50th term of this sequence.	99

	5 points each	
6	An arithmetic sequence has first term 1 and common difference 2. Find the sum of the first 50 terms of this sequence.	2500
7	Evaluate: $\sum_{n=1}^{25}(20n+4)$	6600
8	In my house of cards, starting from the top, the first row consists of 3 cards, and each other row consists of 2 times as many cards as the row above it. If my house of cards consists of 10 rows, then it consists of how many total cards?	3069
9	Suppose that a sequence $\{a_n\}$ is defined by $a_n = an^2 + bn + c$, where *a*, *b*, and *c* are real numbers, and suppose that the first three terms of this sequence are $a_1 = 2, a_2 = 5$, and $a_3 = 11$. Find the fourth term a_4 of this sequence.	20
10	Let *i* be the imaginary unit. Find the 2015th term of the sequence $\{i^n\}_{n=1}^{\infty}$, written in $a+bi$ form.	$-i$

6 points each		
11	Let i be the imaginary unit, and let $a_n = i^n$, where n is a natural number. Define a new sequence $\{s_n\}_{n=1}^{\infty}$ by $s_n = \sum_{i=1}^{n} a_i$. Find the value of s_{2015}.	-1
12	For a sequence $\{a_i\}_{i=1}^{\infty}$, the sum of the first n terms of this sequence is equal to $n^2 + 2n$. Find the numerical value of a_1.	3
13	A radioactive substance initially weighs 1224 grams and has a half-life of 1 day. What is the weight, in grams, of this substance exactly 3 days after it was initially weighed?	153
14	Evaluate: $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 3}{3^n}$	$\frac{15}{2}$ or 7.5
15	A sequence of increasing integers is defined in the following way: any term is the geometric mean of the term before it and the term after it. If the first term of this sequence is 3, find the least possible value of the fourth term of this sequence.	24

2015 – 2016 Log1 Contest Round 2
Alpha Sequences & Series

Name: _______________

	4 points each	
1	Find the 2016th term of the sequence $1,\ 2,\ ...,\ n,\ ...$.	2016
2	How many triangles exist whose sides have lengths equal to three consecutive numbers from the Fibonacci sequence?	0
3	Consider the sequence defined recursively by $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. If $a_1 = \ln 2$ and $a_2 = \ln 3$, find the value of a_7.	$\ln 209952$
4	Consider the sequence whose *n*th term is given by $a_n = \cos\left(\frac{n\pi}{6}\right) + i\sin\left(\frac{n\pi}{6}\right)$, where i is the imaginary unit. Find a_{2015}, written in $a+bi$ form, where *a* and *b* are real numbers.	$\frac{\sqrt{3}}{2} - \frac{1}{2}i$
5	An arithmetic sequence has first term 1 and common difference 2. Find the sum of the first 50 terms of this sequence.	2500

	5 points each	
6	In my house of cards, starting from the top, the first row consists of 2 cards, and each other row consists of 3 times as many cards as the row above it. If my house of cards consists of 10 rows, then it consists of how many total cards?	59048
7	Evaluate: $\sum_{n=1}^{\infty} \frac{2}{n^2 + 2n}$	$\frac{3}{2}$ or 1.5
8	Suppose that a sequence $\{a_n\}$ is defined by $a_n = an^2 + bn + c$, where *a*, *b*, and *c* are real numbers, and suppose that the first three terms of this sequence are $a_1 = 2, a_2 = 5$, and $a_3 = 11$. Find the fourth term a_4 of this sequence.	20
9	Let *i* be the imaginary unit, and let $a_n = i^n$, where *n* is a natural number. Define a new sequence $\{s_n\}_{n=1}^{\infty}$ by $s_n = \sum_{i=1}^{n} a_i$. Find the value of s_{2015}.	-1
10	For the sequence whose *n*th term is $a_n = \binom{n}{3}$, how many of the first 100 terms are divisible by 3?	34

6 points each		
11	For a sequence $\{a_i\}_{i=1}^{\infty}$, the sum of the first *n* terms of this sequence is equal to $\frac{4}{n^2+2n}$. Find the numerical value of a_2.	$-\frac{5}{6}$
12	A radioactive substance initially weighs 1224 grams and has a half-life of 1 day. What is the weight, in grams, of this substance exactly 3 days after it was initially weighed?	153
13	Let $f(x)=3x^4-12x^3+14x^2+15x-10$, and let $g_1(x)=x-1$. When *f* is divided by g_1 according to the Division Algorithm, the quotient is $q_1(x)$ and the remainder is real number r_1. For integer $n\geq 2$, let r_n be the remainder provided by the Division Algorithm when $q_{n-1}(x)$ is divided by $g_n(x)=x-n$. Find the numerical value of r_3.	17
14	A sequence of increasing integers is defined in the following way: any term is the geometric mean of the term before it and the term after it. If the first term of this sequence is 3, find the least possible value of the fourth term of this sequence.	24
15	Let $F_1=F_2=1$, and for integer $n\geq 3$, $F_n=F_{n-1}+F_{n-2}$. As *n* grows without bound, the ratio $\frac{F_n}{F_{n-1}}$ tends toward what number?	$\frac{1+\sqrt{5}}{2}$

2015 – 2016 Log1 Contest Round 2
Mu Sequences & Series

Name: ________________

	4 points each	
1	Find the 2016th term of the sequence $1, 2, ..., n, ...$.	2016
2	How many triangles exist whose sides have lengths equal to three consecutive numbers from the Fibonacci sequence?	0
3	Evaluate: $\sum_{n=1}^{\infty} \frac{n+3}{2^n}$	5
4	Consider the sequence whose *n*th term is given by $a_n = \cos\left(\frac{n\pi}{6}\right) + i\sin\left(\frac{n\pi}{6}\right)$, where i is the imaginary unit. Find a_{2015}, written in $a+bi$ form, where *a* and *b* are real numbers.	$\frac{\sqrt{3}}{2} - \frac{1}{2}i$
5	Consider the sequence whose *n*th term is given by $a_n = \int_0^{\frac{n\pi}{6}} \sin x \, dx$. Find the value of a_8.	$\frac{3}{2}$ or 1.5

	5 points each	
6	In my house of cards, starting from the top, the first row consists of 2 cards, and each other row consists of 3 times as many cards as the row above it. If my house of cards consists of 10 rows, then it consists of how many total cards?	59048
7	Evaluate: $\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$	$\frac{3}{2}$ or 1.5
8	Suppose that a sequence $\{a_n\}$ is such that a_n is a polynomial expression in *n* with real coefficients. Further, suppose that the first four terms of this sequence are $a_1 = 2, a_2 = 7, a_3 = 14$, and $a_4 = 25$, and the degree of a_n is as small as possible. Find the fifth term a_5 of this sequence.	42
9	Let *i* be the imaginary unit, and let $a_n = i^n$, where *n* is a natural number. Let $s_n = \sum_{i=1}^{n} a_i$ and define a new sequence $\{b_n\}_{n=1}^{\infty}$ by $b_n = \sum_{i=1}^{n} s_i$. Find the value of b_{2015}, written in $a+bi$ form.	-1008 $+1008i$
10	For a sequence $\{a_i\}_{i=1}^{\infty}$, the sum of the first *n* terms of this sequence is equal to $\frac{4}{n^2+2n}$. Find the numerical value of a_2.	$-\frac{5}{6}$

6 points each		
11	Let $f(x)=15x^5-14x^4+9x^2-22x+13$, and define a sequence $\{a_n\}_{n=1}^{\infty}$ by $a_n=f^{(n)}(2)$. Find the numerical value of a_4.	3264
12	A radioactive substance initially weighs 1224 grams and has a half-life of 1 day. What is the weight, in grams, of this substance exactly 3 days after it was initially weighed?	153
13	Let $f(x)=3x^4-12x^3+14x^2+15x-10$, and let $g_1(x)=x-1$. When f is divided by g_1 according to the Division Algorithm, the quotient is $q_1(x)$ and the remainder is real number r_1. For integer $n\geq 2$, let r_n be the remainder provided by the Division Algorithm when $q_{n-1}(x)$ is divided by $g_n(x)=x-n$. Find the numerical value of r_3.	17
14	A sequence of increasing integers is defined in the following way: any term is the geometric mean of the term before it and the term after it. If the first term of this sequence is 3, find the least possible value of the fourth term of this sequence.	24
15	Let $F_1=F_2=1$, and for integer $n\geq 3$, $F_n=F_{n-1}+F_{n-2}$. Find the value of $\lim_{n\to\infty}\frac{F_n}{F_{n-1}}$.	$\frac{1+\sqrt{5}}{2}$

2015 – 2016 Log1 Contest Round 2
Sequences & Series Solutions

Mu	Al	Th	Solution
1	1	1	The *n*th term is *n*, so the 2016th term is 2016.
2	2	2	Since the lesser two numbers' sum is the greatest number, the three numbers would not satisfy the triangle inequality. Therefore, there are 0 such triangles.
		3	Every 3*n*th term in the sequence is a 3, and since 2016 is divisible by 3, the 2016th term of the sequence is 3.
	3	4	Count the number of ways of rolling a sum of 4 or less, since there are fewer of those. The combinations of rolls are 1 and 1 (1 way), 1 and 2 (2 ways), 1 and 3 (2 ways), and 2 and 2 (1 way), making a total of six possible rolls with a sum of 4 or less. The total number of rolls is $20^2=400$, so the Sequences & Series is $1-\frac{6}{400}=\frac{394}{400}=\frac{197}{200}$.
3			Let S be the sum of the series. Then $S=\frac{4}{2}+\frac{5}{4}+\frac{6}{8}+\frac{7}{16}+\ldots$. Multiply both sides of this equation by $\frac{1}{2}$, then subtract from the equation to get $\frac{1}{2}S=\frac{4}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$. On the right-hand side of this new equation, beginning with $\frac{1}{4}$, this is an infinite geometric series with common ratio $\frac{1}{2}$. Therefore, $\frac{1}{2}S=\frac{4}{2}+\frac{\frac{1}{4}}{1-\frac{1}{2}}=\frac{4}{2}+\frac{1}{2}=\frac{5}{2}\Rightarrow S=5$.
4	4		$a_{2015}=\cos\left(\frac{2015\pi}{6}\right)+i\sin\left(\frac{2015\pi}{6}\right)=\cos\left(\frac{11\pi}{6}\right)+i\sin\left(\frac{11\pi}{6}\right)=\frac{\sqrt{3}}{2}-\frac{1}{2}i$
		5	$a_{50}=a_1+(50-1)d=1+49\cdot 2=1+98=99$
	5	6	$S_{50}=\frac{50}{2}\left(2a_1+(50-1)d\right)=25(2\cdot 1+49\cdot 2)=25\cdot 100=2500$
5			$a_8=\int_0^{\frac{8\pi}{6}}\sin x\,dx=-\cos x\Big\vert_0^{\frac{8\pi}{6}}=-\cos\frac{8\pi}{6}+\cos 0=-\left(-\frac{1}{2}\right)+1=\frac{3}{2}$
6	6		This is a geometric series with first term 2 and tenth term $2\cdot 3^9=39366$. Therefore, the sum is $\frac{2-3\cdot 39366}{1-3}=59048$.
		7	$\sum_{n=1}^{25}(20n+4)=\frac{25}{2}(24+504)=6600$
7	7		$\sum_{n=1}^{\infty}\frac{2}{n^2+2n}=\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)=\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\ldots$, and the only terms that remain after cancelling are the 1 in the first term and the $\frac{1}{2}$ in the second term, so the sum is $1+\frac{1}{2}=\frac{3}{2}$.

		8	This is a geometric series with first term 3 and tenth term $3\cdot 2^9=1536$. Therefore, the sum is $\frac{3-2\cdot 1536}{1-2}=3069$.
	8	9	Using the method of finite differences, $2 \quad 5 \quad 11 \quad f(4)$ $3 \quad 6 \quad f(4)-11$ $3 \quad f(4)-17$ Since the function is quadratic, $f(4)-17=3\Rightarrow f(4)=20$.
8			Using the method of finite differences, $2 \quad 7 \quad 14 \quad 25 \quad f(5)$ $5 \quad 7 \quad 11 \quad f(5)-25$ $2 \quad 4 \quad f(5)-36$ $2 \quad f(5)-40$ Since the function has degree as small as possible, the data are consistent with a cubic (or higher degree) polynomial, and $f(5)-40=2\Rightarrow f(5)=42$.
		10	The 2015th term is $i^{2015}=i^3=-i$.
	9	11	Every four terms $(i,-1,-i,1)$ will cancel when added, so the sum is just $i+(-1)+(-i)$ $=-1$.
9			The sequence of terms for s_n is $i,-1+i,-1,0,\ldots$ so every four terms added yields $-2+2i$. 503 of these are added, plus the first three terms of s_n, so the sum is $503(-2+2i)+i+(-1+i)+(-1)=-1008+1008i$.
	10		First, $a_1=a_2=0$ since it is impossible to choose more objects than are available (and those two numbers are divisible by 3). For $n\geq 3$, $a_n=\binom{n}{3}=\frac{n(n-1)(n-2)}{3\cdot 2\cdot 1}$. Only one of the factors in the numerator can be divisible by 3, but if only one factor of 3 is available, it will cancel with the factor of 3 in the denominator. Therefore, we need *n*, $n-1$, or $n-2$ to be divisible by 9. There are 11 values where *n* is divisible by 9, 11 values where $n-1$ is divisible by 9, and 10 values where $n-2$ is divisible by 9 (101 would have been the 11th value, but we were only considering the first 100 terms). Therefore, there are $2+11+11+10=34$ terms that are divisible by 3.
10	11		Let s_n be the sum of the first *n* terms of the sequence. Then $a_2=s_2-s_1=\frac{4}{2^2+2\cdot 2}$ $-\frac{4}{1^2+2\cdot 1}=\frac{1}{2}-\frac{4}{3}=-\frac{5}{6}$.
11			$f'(x)=75x^4-56x^3+18x-22$, $f''(x)=300x^3-168x^2+18$, $f'''(x)=900x^2-336x$, and $f^{(4)}(x)=1800x-336$, so $a_4=f^{(4)}(2)=1800\cdot 2-336=3264$.
12	12	13	The weight, in grams, at exactly *n* days after the initial weight is given by $m_n=1224\left(\frac{1}{2}\right)^n$, so the weight after three days is $m_3=1224\left(\frac{1}{2}\right)^3=153$ grams.

13	13		By the Division Algorithm, $3x^4-12x^3+14x^2+15x-10=(x-1)(3x^3-9x^2+5x+20)+10$ $3x^3-9x^2+5x+20=(x-2)(3x^2-3x-1)+18$ $3x^2-3x-1=(x-3)(3x+6)+17$, so the third remainder is 17.
		14	Let S be the sum of the series. Then $S=\frac{10}{3}+\frac{19}{9}+\frac{30}{27}+\frac{43}{81}+\frac{58}{243}+\ldots$. Multiply both sides of this equation by $\frac{1}{3}$, then subtract from the equation to get $\frac{2}{3}S=\frac{10}{3}+\frac{9}{9}+\frac{11}{27}+\frac{13}{81}+\frac{15}{243}+\ldots$. Multiply both sides of this equation by $\frac{1}{3}$, then subtract from that equation to get $\frac{4}{9}S=\frac{10}{3}-\frac{1}{9}+\frac{2}{27}+\frac{2}{81}+\frac{2}{243}+\ldots$. On the right-hand side of this new equation, beginning with $\frac{2}{27}$, this is an infinite geometric series with common ratio $\frac{1}{3}$. Therefore, $\frac{4}{9}S=\frac{10}{3}-\frac{1}{9}+\frac{\frac{2}{27}}{1-\frac{1}{3}}=\frac{10}{3}-\frac{1}{9}+\frac{1}{9}=\frac{10}{3}\Rightarrow S=\frac{10}{3}\cdot\frac{9}{4}=\frac{15}{2}$.
14	14	15	Let the sequence's first three terms 3, *a*, and *b*. Then $a^2=3b$, meaning that *a* is divisible by 3. This means that $3b=(3a_1)^2=9a_1^2\Rightarrow b=3a_1^2$ for some integer a_1. To make this as small as possible but still increasing, make $a_1=2\Rightarrow b=12\Rightarrow a=6$, so the smallest possible first three terms are 3, 6, 12. To keep these numbers as small as possible, maintain a common ratio of 2, making the least possible fourth term 24.
15	15		$\frac{F_n}{F_{n-1}}=\frac{F_{n-1}+F_{n-2}}{F_{n-1}}=1+\frac{F_{n-2}}{F_{n-1}}$, so if *F* is the number to which the sequence tends (its limit), *F* satisfies the equation $F=1+\frac{1}{F}\Rightarrow F^2=F+1\Rightarrow F^2-F-1=0$, and since the Fibonacci numbers are all positive, *F* must be the positive number that satisfies this equation. Therefore, $F=\frac{-(-1)+\sqrt{(-1)^2-4\cdot1\cdot(-1)}}{2}=\frac{1+\sqrt{5}}{2}$.