

2015 - 2016 Log1 Contest Round 1
Theta Functions

Name: _____

4 points each		
1	Find the value of $f(-3)$, given that $f(x) = -5x - 17$.	
2	If $f(x, y) = \frac{x-y}{y}$, find the positive real value of b such that $f(b, 8) = f(2, b)$.	
3	If $f(x) = - x $, find the value of $(f \circ f)(2)$.	
4	Find the value of $f(5)$, given that $f(x) = -4x^5 + 19x^4 - 2x^2 - x + 156$.	
5	If $f(x) = \sqrt{x+2}$, find the value of $f^{-1}(3)$.	

5 points each		
6	What is the domain, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?	
7	What is the range, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?	
8	If the domain of real-valued function $f(x)$ is $[-3, 6]$, what is the domain of $f(5 - x^3)$, written in interval notation?	
9	A circle is centered at the origin and has radius of length 5. The tangent $y = f(x)$ to this circle at the point $(-4, -3)$ is drawn. Find the value of $f(0)$.	
10	Suppose $A(P, n, r, t)$ represents the total amount of money accumulated in t years in an account in which $\$P$ is initially invested, with interest compounded n times per year at $r\%$ annual interest. Find the value of $A(1000, 2, 4, 2)$, written in dollars and cents, rounded to the nearest cent.	

6 points each

11	Is the function $f(x) = 3x^3 + \frac{x}{x^2+1} + \frac{1}{x}$ even (meaning $f(x) = f(-x)$ for all x in the domain of f), odd (meaning $f(x) = -f(-x)$ for all x in the domain of f), both, or neither?	
12	Let $f(n)$ be the number of positive integral divisors of the positive integer n . Find the value of $f(2015)$.	
13	Let $f(x) = 3x^8 - 2x^7 + 9x^6 - x^5 + 3x^3 + 17x^2 - 2x + 5$ and $g(x) = 4x - 3$. If $q(x)$ and $r(x)$ are the quotient and remainder functions, respectively, that exist according to the Division Algorithm when $f(x)$ is divided by $g(x)$, find the value of $q(1) + r(2015)$.	
14	Define a function $T: \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ by $T(x, y, z) = \begin{cases} y, & \text{if } x \leq y \\ g(x, y, z), & \text{otherwise} \end{cases}$, and let $g: \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ be defined as $g(x, y, z) = \begin{cases} z, & \text{if } y \leq z \\ x, & \text{otherwise} \end{cases}$. Find the value of $T(14, 11, 18)$.	
15	Let $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ be a look-and-say function, defined by $f(n)$ = the number of each type of digit of n when reading n from left to right. For example, $f(1) = 11$ ("one one"), $f(33) = 23$ ("two threes"), and $f(121) = 111211$ ("one one, one two, one one"). Find the value of $f(12231)$.	

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Alpha Functions

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4 points each	
1	Find the value of $f(-3)$, given that $f(x) = -5x - 17$.
2	If $f(x, y) = \frac{x-y}{y}$, find the positive real value of b such that $f(b, 8) = f(2, b)$.
3	If $f(x) = - x $, find the value of $(f \circ f)(2)$.
4	Find the value of $f(5)$, given that $f(x) = -4x^5 + 19x^4 - 2x^2 - x + 156$.
5	If $f(x, y) = f(x+y, x)$ and $f(0, 1) = 2$, how many of the following five function values must also equal 2? $f(2, 1)$ $f(1, 2)$ $f(8, 5)$ $f(1, 1)$ $f(-1, 2)$

5 points each	
6	What is the domain, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?
7	What is the range, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?
8	If the domain of real-valued function $f(x)$ is $[-3, 6]$, what is the domain of $f(5 - (x-1)^3)$, written in interval notation?
9	A circle is centered at the origin and has radius of length 5. The tangent $y = f(x)$ to this circle at the point $(-4, -3)$ is drawn. Find the value of $f(2015)$.
10	Suppose $A(P, n, r, t)$ represents the total amount of money accumulated in t years in an account in which $\$P$ is initially invested, with interest compounded n times per year at $r\%$ annual interest. Find the value of $A(1000, 2, 4, 2)$, written in dollars and cents, rounded to the nearest cent.

6 points each

11	Is the function $f(x) = 3x^3 + \frac{x}{x^2+1} + \frac{1}{x}$ even, odd, both, or neither?	
12	Let $f(n)$ be the sum of the positive integral divisors of the positive integer n . Find the value of $f(2015)$.	
13	Let $f(x) = 3x^8 - 2x^7 + 9x^6 - x^5 + 3x^3 + 17x^2 - 2x + 5$ and $g(x) = 4x - 3$. If $q(x)$ and $r(x)$ are the quotient and remainder functions, respectively, that exist according to the Division Algorithm when $f(x)$ is divided by $g(x)$, find the value of $q(1) + r(2015)$.	
14	For positive integer n , let $f(n)$ be the magic constant of a normal $n \times n$ magic square. Find the greatest value of n such that $f(n) < 2015$.	
15	Let $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ be a look-and-say function, defined by $f(n)$ = the number of each type of digit of n when reading n from left to right. For example, $f(1) = 11$ ("one one"), $f(33) = 23$ ("two threes"), and $f(121) = 111211$ ("one one, one two, one one"). Find the value of $f(12231)$.	

2015 - 2016 Log1 Contest Round 1
Mu Functions

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4 points each		
1	Find the value of $f(-3)$, given that $f(x) = -5x - 17$.	
2	Find the value of $f'(3)$, given that $f(x) = -5x - 17$.	
3	If $f(x) = - x $, find the value of $(f \circ f)(2)$.	
4	Find the value of $f(5)$, given that $f(x) = -4x^5 + 19x^4 - 2x^2 - x + 156$.	
5	If $f'(x) = \frac{1}{x} + \frac{1}{1+x^2}$ and $f(1) = \frac{\pi}{2}$, find the value of $f(-1)$.	

5 points each		
6	What is the domain, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?	
7	The function $f(x) = x^3 - 3x^2 + 4$ has domain of all real numbers, but the inverse relation of this function is itself not a function. If some interval(s) of the form $[a, b]$, where $a < b$, were to be removed from the domain of f in order to make the inverse relation a function, determine the least possible length of that/those interval(s).	
8	A circle is centered at the origin and has radius of length 5. The tangent $y = f(x)$ to this circle at the point $(-4, -3)$ is drawn. Find the value of $f(2015)$.	
9	Find the value(s) of x in the interval $[-2, 3]$ that satisfy the Mean Value Theorem for the function $x^3 - x^2 + x - 3$.	
10	Find the intervals on which the function $f(x) = 3x^4 - 20x^3 + 12x^2 + 96x - 18$ is increasing.	

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11	Is the function $f(x) = 3x^3 + \frac{x}{x^2+1} + \frac{1}{x}$ even, odd, both, or neither?	
12	Let $f(x) = 3x^8 - 2x^7 + 9x^6 - x^5 + 3x^3 + 17x^2 - 2x + 5$ and $g(x) = 4x - 3$. If $q(x)$ and $r(x)$ are the quotient and remainder functions, respectively, that exist according to the Division Algorithm when $f(x)$ is divided by $g(x)$, find the value of $q(1) + r(2015)$.	
13	Consider the function $f(x) = x^3 + 3x^2 - 24x - 3$. If M and m are the maximum and minimum values, respectively, of f on the interval $[-5, -2]$, find the value of $M - m$.	
14	There is a point on the function $f(x) = x^3 + 2x^2 - 20x + 24$ such that the tangent to f at that point is the x -axis. What is that point?	
15	Let $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ be a look-and-say function, defined by $f(n)$ = the number of each type of digit of n when reading n from left to right. For example, $f(1) = 11$ ("one one"), $f(33) = 23$ ("two threes"), and $f(121) = 111211$ ("one one, one two, one one). Find the value of $f(12231)$.	

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Theta Functions

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4 points each		
1	Find the value of $f(-3)$, given that $f(x) = -5x - 17$.	-2
2	If $f(x, y) = \frac{x-y}{y}$, find the positive real value of b such that $f(b, 8) = f(2, b)$.	4
3	If $f(x) = - x $, find the value of $(f \circ f)(2)$.	-2
4	Find the value of $f(5)$, given that $f(x) = -4x^5 + 19x^4 - 2x^2 - x + 156$.	-524
5	If $f(x) = \sqrt{x+2}$, find the value of $f^{-1}(3)$.	7

5 points each		
6	What is the domain, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?	[-1,5]
7	What is the range, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?	[0,3]
8	If the domain of real-valued function $f(x)$ is $[-3, 6]$, what is the domain of $f(5 - x^3)$, written in interval notation?	[-1,2]
9	A circle is centered at the origin and has radius of length 5. The tangent $y = f(x)$ to this circle at the point $(-4, -3)$ is drawn. Find the value of $f(0)$.	$-\frac{25}{3}$
10	Suppose $A(P, n, r, t)$ represents the total amount of money accumulated in t years in an account in which $\$P$ is initially invested, with interest compounded n times per year at $r\%$ annual interest. Find the value of $A(1000, 2, 4, 2)$, written in dollars and cents, rounded to the nearest cent.	\$1082.43

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11	Is the function $f(x) = 3x^3 + \frac{x}{x^2+1} + \frac{1}{x}$ even (meaning $f(x) = f(-x)$ for all x in the domain of f), odd (meaning $f(x) = -f(-x)$ for all x in the domain of f), both, or neither?	odd
12	Let $f(n)$ be the number of positive integral divisors of the positive integer n . Find the value of $f(2015)$.	8
13	Let $f(x) = 3x^8 - 2x^7 + 9x^6 - x^5 + 3x^3 + 17x^2 - 2x + 5$ and $g(x) = 4x - 3$. If $q(x)$ and $r(x)$ are the quotient and remainder functions, respectively, that exist according to the Division Algorithm when $f(x)$ is divided by $g(x)$, find the value of $q(1) + r(2015)$.	32
14	Define a function $T: \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ by $T(x, y, z) = \begin{cases} y, & \text{if } x \leq y \\ g(x, y, z), & \text{otherwise} \end{cases}$, and let $g: \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ be defined as $g(x, y, z) = \begin{cases} z, & \text{if } y \leq z \\ x, & \text{otherwise} \end{cases}$. Find the value of $T(14, 11, 18)$.	18
15	Let $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ be a look-and-say function, defined by $f(n)$ = the number of each type of digit of n when reading n from left to right. For example, $f(1) = 11$ ("one one"), $f(33) = 23$ ("two threes"), and $f(121) = 111211$ ("one one, one two, one one"). Find the value of $f(12231)$.	11221311

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4 points each		
1	Find the value of $f(-3)$, given that $f(x) = -5x - 17$.	-2
2	If $f(x, y) = \frac{x-y}{y}$, find the positive real value of b such that $f(b, 8) = f(2, b)$.	4
3	If $f(x) = - x $, find the value of $(f \circ f)(2)$.	-2
4	Find the value of $f(5)$, given that $f(x) = -4x^5 + 19x^4 - 2x^2 - x + 156$.	-524
5	If $f(x, y) = f(x+y, x)$ and $f(0, 1) = 2$, how many of the following five function values must also equal 2? $f(2, 1)$ $f(1, 2)$ $f(8, 5)$ $f(1, 1)$ $f(-1, 2)$	4

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6	What is the domain, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?	[-1, 5]
7	What is the range, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?	[0, 3]
8	If the domain of real-valued function $f(x)$ is $[-3, 6]$, what is the domain of $f(5 - (x-1)^3)$, written in interval notation?	[-1, 3]
9	A circle is centered at the origin and has radius of length 5. The tangent $y = f(x)$ to this circle at the point $(-4, -3)$ is drawn. Find the value of $f(2015)$.	-2695
10	Suppose $A(P, n, r, t)$ represents the total amount of money accumulated in t years in an account in which $\$P$ is initially invested, with interest compounded n times per year at $r\%$ annual interest. Find the value of $A(1000, 2, 4, 2)$, written in dollars and cents, rounded to the nearest cent.	\$1082.43

6 points each

11	Is the function $f(x) = 3x^3 + \frac{x}{x^2+1} + \frac{1}{x}$ even, odd, both, or neither?	odd
12	Let $f(n)$ be the sum of the positive integral divisors of the positive integer n . Find the value of $f(2015)$.	2,688
13	Let $f(x) = 3x^8 - 2x^7 + 9x^6 - x^5 + 3x^3 + 17x^2 - 2x + 5$ and $g(x) = 4x - 3$. If $q(x)$ and $r(x)$ are the quotient and remainder functions, respectively, that exist according to the Division Algorithm when $f(x)$ is divided by $g(x)$, find the value of $q(1) + r(2015)$.	32
14	For positive integer n , let $f(n)$ be the magic constant of a normal $n \times n$ magic square. Find the greatest value of n such that $f(n) < 2015$.	15
15	Let $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ be a look-and-say function, defined by $f(n)$ = the number of each type of digit of n when reading n from left to right. For example, $f(1) = 11$ ("one one"), $f(33) = 23$ ("two threes"), and $f(121) = 111211$ ("one one, one two, one one"). Find the value of $f(12231)$.	11221311

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4 points each		
1	Find the value of $f(-3)$, given that $f(x) = -5x - 17$.	-2
2	Find the value of $f'(3)$, given that $f(x) = -5x - 17$.	-5
3	If $f(x) = - x $, find the value of $(f \circ f)(2)$.	-2
4	Find the value of $f(5)$, given that $f(x) = -4x^5 + 19x^4 - 2x^2 - x + 156$.	-524
5	If $f'(x) = \frac{1}{x} + \frac{1}{1+x^2}$ and $f(1) = \frac{\pi}{2}$, find the value of $f(-1)$.	0

5 points each		
6	What is the domain, written in interval notation, of the real-valued function $f(x) = \sqrt{9 - (x-2)^2}$?	[-1,5]
7	The function $f(x) = x^3 - 3x^2 + 4$ has domain of all real numbers, but the inverse relation of this function is itself not a function. If some interval(s) of the form $[a, b]$, where $a < b$, were to be removed from the domain of f in order to make the inverse relation a function, determine the least possible length of that/those interval(s).	2
8	A circle is centered at the origin and has radius of length 5. The tangent $y = f(x)$ to this circle at the point $(-4, -3)$ is drawn. Find the value of $f(2015)$.	-2695
9	Find the value(s) of x in the interval $[-2, 3]$ that satisfy the Mean Value Theorem for the function $x^3 - x^2 + x - 3$.	$\frac{1 \pm \sqrt{19}}{3}$
10	Find the intervals on which the function $f(x) = 3x^4 - 20x^3 + 12x^2 + 96x - 18$ is increasing.	$(-1, 2) \cup (4, \infty)$

6 points each

11	Is the function $f(x) = 3x^3 + \frac{x}{x^2+1} + \frac{1}{x}$ even, odd, both, or neither?	odd
12	Let $f(x) = 3x^8 - 2x^7 + 9x^6 - x^5 + 3x^3 + 17x^2 - 2x + 5$ and $g(x) = 4x - 3$. If $q(x)$ and $r(x)$ are the quotient and remainder functions, respectively, that exist according to the Division Algorithm when $f(x)$ is divided by $g(x)$, find the value of $q(1) + r(2015)$.	32
13	Consider the function $f(x) = x^3 + 3x^2 - 24x - 3$. If M and m are the maximum and minimum values, respectively, of f on the interval $[-5, -2]$, find the value of $M - m$.	28
14	There is a point on the function $f(x) = x^3 + 2x^2 - 20x + 24$ such that the tangent to f at that point is the x -axis. What is that point?	(2,0)
15	Let $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ be a look-and-say function, defined by $f(n)$ = the number of each type of digit of n when reading n from left to right. For example, $f(1) = 11$ ("one one"), $f(33) = 23$ ("two threes"), and $f(121) = 111211$ ("one one, one two, one one). Find the value of $f(12231)$.	11221311

**2015 - 2016 Log1 Contest Round 1
Functions Solutions**

Mu	Al	Th	Solution
1	1	1	$f(-3) = -5(-3) - 17 = 15 - 17 = -2$
	2	2	$\frac{b-8}{8} = \frac{2-b}{b} \Rightarrow b^2 - 8b = 16 - 8b \Rightarrow b^2 = 16$, so since $b > 0$, $b = 4$.
2			Since $f'(x) = -5$ for all x , $f'(3) = -5$.
3	3	3	$(f \circ f)(2) = f(f(2)) = f(-2) = - -2 = -2$
4	4	4	$f(5) = -4(5)^5 + 19(5)^4 - 2(5)^2 - 5 + 156 = -12500 + 11875 - 50 - 5 + 156 = -524$ (admittedly, this is probably easier to do with synthetic division)
		5	$3 = \sqrt{f^{-1}(3) + 2} \Rightarrow 9 = f^{-1}(3) + 2 \Rightarrow 7 = f^{-1}(3)$
	5		Replacing the x -coordinate with the sum of the x - and y -coordinates and the y -coordinate with the x -coordinate, the following function values are all equal to 2: ..., $f(2, -3)$, $f(-1, 2)$, $f(1, -1)$, $f(0, 1)$, $f(1, 0)$, $f(1, 1)$, $f(2, 1)$, $f(3, 2)$, $f(5, 3)$, $f(8, 5)$, ... Extending the list to the left, the signs of the two coordinates will be of opposite signs, so $f(1, 2)$ will never appear in that direction. Extending the list to the right, we continue to get consecutive terms in the Fibonacci sequence for the inputs, so $f(1, 2)$ will never appear in that direction either. The other four terms are listed above, so the answer is 4.
5			$f'(x) = \frac{1}{x} + \frac{1}{1+x^2} \Rightarrow f(x) = \ln x + \arctan x + c \Rightarrow \frac{\pi}{2} = \ln 1 + \arctan 1 + c = \frac{\pi}{4} + c \Rightarrow c = \frac{\pi}{4}$. Therefore, $f(x) = \ln x + \arctan x + \frac{\pi}{4} \Rightarrow f(-1) = \ln 1 + \arctan(-1) + \frac{\pi}{4} = 0 - \frac{\pi}{4} + \frac{\pi}{4} = 0$.
6	6	6	$9 - (x-2)^2 \geq 0 \Rightarrow (x-2)^2 \leq 9 \Rightarrow -3 \leq x-2 \leq 3 \Rightarrow -1 \leq x \leq 5$, so the interval is $[-1, 5]$.
	7	7	The graph of the function is the upper half of a circle centered at $(2, 0)$ with radius of length 3, so the range must be $[0, 3]$.
7			The function has zeros at -1 and 2 , a relative maximum at $(0, 4)$, and a relative minimum at $(2, 0)$. Further, the other point with a y -value of 4 is $(3, 4)$, so the portion of the domain of f with multiple points with the same y -values is $[-1, 3]$; further, if this interval were broken up into three intervals with non-repeating y -values, those three intervals would be $[-1, 0]$, $[0, 2]$, and $[2, 3]$. Removing any two of those intervals would result in an inverse function, and removing the outer two would be the way to do that resulting in the least length of such an interval. The total length of those two intervals is 2.
		8	$-3 \leq 5 - x^3 \leq 6 \Rightarrow -8 \leq -x^3 \leq 1 \Rightarrow -1 \leq x^3 \leq 8 \Rightarrow -1 \leq x \leq 2$, so the interval is $[-1, 2]$.
	8		$-3 \leq 5 - (x-1)^3 \leq 6 \Rightarrow -8 \leq -(x-1)^3 \leq 1 \Rightarrow -1 \leq (x-1)^3 \leq 8 \Rightarrow x-1 \leq 2 \Rightarrow -2 \leq x-1 \leq 2 \Rightarrow -1 \leq x \leq 3$, so the interval is $[-1, 3]$.

		9	The slope of the radius drawn to this point is $\frac{-3-0}{-4-0} = \frac{3}{4}$, so the slope of the tangent at that point is $-\frac{4}{3}$. This makes the equation of the tangent line $y = -\frac{4}{3}(x+4) - 3$, and therefore, $f(0) = -\frac{4}{3}(4) - 3 = -\frac{25}{3}$.
8	9		The slope of the radius drawn to this point is $\frac{-3-0}{-4-0} = \frac{3}{4}$, so the slope of the tangent at that point is $-\frac{4}{3}$. This makes the equation of the tangent line $y = -\frac{4}{3}(x+4) - 3$, and therefore, $f(2015) = -\frac{4}{3}(2019) - 3 = -2695$.
9			Since f is a polynomial, it is continuous and differentiable everywhere. Therefore, we are trying to solve $f'(x) = \frac{f(3) - f(-2)}{3 - (-2)} = \frac{18 - (-17)}{5} = 7$ for $x \in [-2, 3]$. Therefore, $3x^2 - 2x + 1 = 7 \Rightarrow 3x^2 - 2x - 6 = 0 \Rightarrow x = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 3 \cdot -6}}{2 \cdot 3} = \frac{1 \pm \sqrt{19}}{3}$. Since both values are in the interval, they are both answers.
	10	10	$A(1000, 2, 4, 2) = 1000 \left(1 + \frac{.04}{2}\right)^{2 \cdot 2} = 1000(1.02)^4 = 1082.43216$, so the answer is \$1082.43 (rounded to the nearest cent).
10			$f'(x) = 12x^3 - 60x^2 + 24x + 96 = 12(x+1)(x-2)(x-4)$, and $f'(x) > 0$ on the intervals $(-1, 2) \cup (4, \infty)$, so this is where f is increasing.
11	11	11	Since $f(-x) = 3(-x)^3 + \frac{-x}{(-x)^2 + 1} + \frac{1}{-x} = -3x^3 - \frac{x}{x^2 + 1} - \frac{1}{x} = -f(x)$, f is odd.
		12	Since the prime factorization of 2015 is $5 \cdot 13 \cdot 31$, the number of positive integral divisors is $(1+1)(1+1)(1+1) = 8$.
	12		Since the prime factorization of 2015 is $5 \cdot 13 \cdot 31$, the sum of the positive integral divisors is $(5+1)(13+1)(31+1) = 2,688$.
12	13	13	Since g is a first-degree polynomial, r is a constant function. Since $\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$, $f(1) = \frac{f(1)}{4(1)-3} = \frac{f(1)}{g(1)} = q(1) + \frac{r(1)}{g(1)} = q(1) + \frac{r(1)}{4(1)-3} = q(1) + r(1) = q(1) + r(2015)$, so $q(1) + r(2015) = f(1) = 32$.
13			$f'(x) = 3x^2 + 6x - 24 = 3(x+4)(x-2)$, so the only critical number in the interval is -4 . Checking the endpoints of the interval as well as this critical number, $f(-5) = 67$, $f(-4) = 77$, and $f(-2) = 49$, so $M - m = 77 - 49 = 28$.
		14	Since $14 > 11$, $T(14, 11, 18) = g(14, 11, 18)$, and since $11 \leq 18$, $g(14, 11, 18) = 18$, so $T(14, 11, 18) = 18$.

	14		<p>A normal $n \times n$ magic square uses the integers from 1 to n^2, inclusive. The sum of these integers is $\frac{n^2(n^2+1)}{2}$. Since each of the n rows has a sum equal to the magic constant, the magic constant is $\frac{\frac{n^2(n^2+1)}{2}}{n} = \frac{n(n^2+1)}{2}$. By inspection, $\frac{15(15^2+1)}{2} = 1695$ and $\frac{16(16^2+1)}{2} = 2056$, so the greatest such value of n is 15.</p>
14			<p>$f(x) = x^3 + 2x^2 - 20x + 24 = (x+6)(x-2)^2$ and $f'(x) = 3x^2 + 4x - 20 = (3x+10)(x-2)$, so the only value satisfying $f(x) = 0$ (meaning the point is on the x-axis) and $f'(x) = 0$ (meaning the tangent is horizontal) is $x = 2$. Therefore, the point is $(2, 0)$.</p>
15	15	15	<p>Since 12231 is one one, two twos, one three, and one one, $f(12231) = 11221311$.</p>