2014 – 2015 Log1 Contest Round 3
Theta Individual

Name: ________________

4 points each		
1	For all integers x, let $d(x)$ be the distance on the number line between x and 2015. If the values of $d(x)$ are written as a sequence in non-decreasing order, find the sum of the first 2015 numbers in the sequence.	
2	Solve for x: $2^{2^3} = 65536^{2^x}$	
3	Convert 731_8 to a base-4 number.	
4	A planar graph consists of 47 edges, each of which connects two each of 30 vertices. No two edges connect the same two vertices directly, and no two edges intersect. Into how many non-overlapping regions does this graph divide the plane?	
5	Baby Björn has a total of $20.09 to his name in the entire world, all in pennies, nickels, dimes, and quarters. Coincidentally, he has the same number of pennies as nickels as dimes as quarters. How many total coins does Baby Björn have in the entire world?	

5 points each		
6	Given a triangle with sides of length 26 meters, 28 meters, and 30 meters, find the length, in meters, of the angle bisector of the triangle to the side with length 28 meters.	
7	Find the area, in square meters, enclosed by an ellipse with major and minor axes of lengths 18 meters and 12 meters, respectively.	
8	If you are dealt three cards from a standard deck of 52 playing cards, what is the probability that the three cards are all of the same suit?	
9	Find the product of the greatest common divisor and the least common multiple of 2015 and 1105.	
10	A regular hexagon and a regular octagon enclose the same area. Find the ratio of the square of the length of a side of the regular octagon to that of the regular hexagon.	

6 points each		
11	Consider the 71 fractions $\frac{1}{72}, \frac{2}{72}, \ldots, \frac{71}{72}$. If each of these fractions is reduced into lowest terms, in how many of these lowest-terms fractions is the numerator equal to 1?	
12	Define a sequence $\langle a_n \rangle_{n=1}^{\infty}$ as $a_1 = 1$ and for any integer $n \geq 2$, $a_n = \dfrac{\sum_{i=1}^{n-1} a_i}{\prod_{i=1}^{n-1} a_i}$. Find the numerical value of a_8.	
13	Find the coefficient of x^5 in the expansion of $(x^3 + 2x^2 - 3)(x-1)(x+1)^2$.	
14	Find the length of the latus rectum of the parabola with equation $y = \frac{x^2}{36} - \frac{x}{6} - \frac{3}{4}$.	
15	A circle has a radius of length $\frac{\sqrt{2}}{2}$ inches. Find the area, in square inches, enclosed by the regular octagon circumscribed about the circle.	

2014 – 2015 Log1 Contest Round 3
Alpha Individual

Name: ________________

	4 points each	
1	For all integers x, let $d(x)$ be the distance on the number line between x and 2015. If the values of $d(x)$ are written as a sequence in non-decreasing order, find the sum of the first 2015 numbers in the sequence.	
2	Convert 731_8 to a base-4 number.	
3	In how many points in the xy-plane do the polar graphs $r=\theta$ and $r=\cos\theta$ intersect?	
4	A planar graph consists of 47 edges, each of which connects two each of 30 vertices. No two edges connect the same two vertices directly, and no two edges intersect. Into how many non-overlapping regions does this graph divide the plane?	
5	Baby Björn has a total of \$20.09 to his name in the entire world, all in pennies, nickels, dimes, and quarters. Coincidentally, he has the same number of pennies as nickels as dimes as quarters. How many total coins does Baby Björn have in the entire world?	

	5 points each	
6	The function $f(x)=-3x^2-12x+14$ has a relative maximum at what point?	
7	Find the area, in square meters, enclosed by an ellipse with major and minor axes of lengths 18 meters and 12 meters, respectively.	
8	If you are dealt three cards from a standard deck of 52 playing cards, what is the probability that the three cards are of consecutive ranks? "Rank" refers to the value on the card: 2, 3, …, Q, K, A. For this problem, count the ace only as a high card—thus, the cards A, 2, and 3 would not be considered as three cards of consecutive ranks, but the cards Q, K, and A would.	
9	Find the product of the greatest common divisor and the least common multiple of 2015 and 1105.	
10	Consider the 71 fractions $\frac{1}{72}, \frac{2}{72}, ..., \frac{71}{72}$. If each of these fractions is reduced into lowest terms, in how many of these lowest-terms fractions is the numerator equal to 1?	

6 points each		
11	Define a sequence $\langle a_n \rangle_{n=1}^{\infty}$ as $a_1 = 1$ and for any integer $n \geq 2$, $a_n = \frac{\sum_{i=1}^{n-1} a_i}{\prod_{i=1}^{n-1} a_i}$. Find the numerical value of a_8.	
12	Let $f(x) = 2x^2 - 1$. Find the values of *x* that satisfy $f(f(x)) = f(x)$.	
13	Find the coefficient of x^5 in the expansion of $(x^3 + 2x^2 - 3)(x-1)(x+1)^2$.	
14	From the endpoints of the latus rectum of the parabola with equation $y = \frac{x^2}{36} - \frac{x}{6} - \frac{3}{4}$, two line segments are drawn perpendicularly to the directrix of the parabola. The latus rectum, the two line segments, and a portion of the directrix form a rectangle that encloses what area?	
15	A circle has a radius of length $2\sqrt{\cot 20^\circ}$ inches. Find the area, in square inches, enclosed by the regular nonagon circumscribed about the circle.	

2014 – 2015 Log1 Contest Round 3
Mu Individual

Name: ________________

	4 points each	
1	For all integers x, let $d(x)$ be the distance on the number line between x and 2015. If the values of $d(x)$ are written as a sequence in non-decreasing order, find the sum of the first 2015 numbers in the sequence.	
2	Consider the graph of $6x^2 - xy - 35y^2 = 0$. At each point on this graph except for the origin, the slope of a tangent line to that point must have one of two distinct slopes. Find the lesser of those two slopes.	
3	Evaluate: $\lim_{x \to -\infty} \frac{3-x}{\sqrt{4x^2+6x+7}}$	
4	A planar graph consists of 47 edges, each of which connects two each of 30 vertices. No two edges connect the same two vertices directly, and no two edges intersect. Into how many non-overlapping regions does this graph divide the plane?	
5	If $f(x) = 4x^3 - \arcsin x$, find the value of $f'\left(\frac{\sqrt{3}}{2}\right)$.	

	5 points each	
6	The function $f(x) = -2x^3 + 5x^2 - 4x - 1$ has a relative maximum at what point?	
7	If $\int_{-5}^{-4} \frac{3x+6}{x^2+3x} dx = \ln\left(\frac{a}{b}\right)$, where a and b are relatively prime positive integers, find the value of $b-a$.	
8	If you are dealt three cards from a standard deck of 52 playing cards, what is the probability that the three cards are of consecutive ranks? "Rank" refers to the value on the card: 2, 3, ..., Q, K, A. For this problem, count the ace only as a high card—thus, the cards A, 2, and 3 would not be considered as three cards of consecutive ranks, but the cards Q, K, and A would.	
9	Consider the 71 fractions $\frac{1}{72}, \frac{2}{72}, ..., \frac{71}{72}$. If each of these fractions is reduced into lowest terms, in how many of these lowest-terms fractions is the numerator equal to 1?	
10	Define a sequence $\langle a_n \rangle_{n=1}^{\infty}$ as $a_1 = 1$ and for any integer $n \geq 2$, $a_n = \frac{\sum_{i=1}^{n-1} a_i}{\prod_{i=1}^{n-1} a_i}$. Find the numerical value of a_8.	

6 points each		
11	Find the average of the positive integral factors of 2015.	
12	Evaluate to the nearest 0.01: $\frac{1}{\sqrt{2\pi}}\int_{-1}^{1} e^{-0.5x^2}\,dx$	
13	Find the coefficient of x^5 in the expansion of $\left(x^3+2x^2-3\right)\left(x-1\right)\left(x+1\right)^2$.	
14	Consider the parabola with equation $y=\frac{x^2}{36}-\frac{x}{6}-\frac{3}{4}$. The tangents to this parabola at the endpoints of its latus rectum intersect at what point?	
15	A circle has a radius of length $\sqrt{2}+\sqrt{6}$ inches. Find the area, in square inches, enclosed by the regular dodecagon circumscribed about the circle.	

2014 – 2015 Log1 Contest Round 3
Theta Individual

Name: ________________

	4 points each	
1	For all integers x, let $d(x)$ be the distance on the number line between x and 2015. If the values of $d(x)$ are written as a sequence in non-decreasing order, find the sum of the first 2015 numbers in the sequence.	1,015,056
2	Solve for x: $2^{2^3} = 65536^{2^x}$	-1
3	Convert 731_8 to a base-4 number.	13121_4 or 13121
4	A planar graph consists of 47 edges, each of which connects two each of 30 vertices. No two edges connect the same two vertices directly, and no two edges intersect. Into how many non-overlapping regions does this graph divide the plane?	19
5	Baby Björn has a total of \$20.09 to his name in the entire world, all in pennies, nickels, dimes, and quarters. Coincidentally, he has the same number of pennies as nickels as dimes as quarters. How many total coins does Baby Björn have in the entire world?	196

	5 points each	
6	Given a triangle with sides of length 26 meters, 28 meters, and 30 meters, find the length, in meters, of the angle bisector of the triangle to the side with length 28 meters.	$3\sqrt{65}$
7	Find the area, in square meters, enclosed by an ellipse with major and minor axes of lengths 18 meters and 12 meters, respectively.	54π
8	If you are dealt three cards from a standard deck of 52 playing cards, what is the probability that the three cards are all of the same suit?	$\frac{22}{425}$
9	Find the product of the greatest common divisor and the least common multiple of 2015 and 1105.	2,226,575
10	A regular hexagon and a regular octagon enclose the same area. Find the ratio of the square of the length of a side of the regular octagon to that of the regular hexagon.	$\frac{3\sqrt{6}-3\sqrt{3}}{4}$ or equiv.

6 points each		
11	Consider the 71 fractions $\frac{1}{72}, \frac{2}{72}, ..., \frac{71}{72}$. If each of these fractions is reduced into lowest terms, in how many of these lowest-terms fractions is the numerator equal to 1?	11
12	Define a sequence $\langle a_n \rangle_{n=1}^{\infty}$ as $a_1 = 1$ and for any integer $n \geq 2$, $a_n = \frac{\sum_{i=1}^{n-1} a_i}{\prod_{i=1}^{n-1} a_i}$. Find the numerical value of a_8.	$\frac{17}{15}$
13	Find the coefficient of x^5 in the expansion of $(x^3 + 2x^2 - 3)(x-1)(x+1)^2$.	3
14	Find the length of the latus rectum of the parabola with equation $y = \frac{x^2}{36} - \frac{x}{6} - \frac{3}{4}$.	36
15	A circle has a radius of length $\frac{\sqrt{2}}{2}$ inches. Find the area, in square inches, enclosed by the regular octagon circumscribed about the circle.	$4\sqrt{2} - 4$ or equiv.

2014 – 2015 Log1 Contest Round 3
Alpha Individual

Name: ________________

4 points each		
1	For all integers *x*, let $d(x)$ be the distance on the number line between x and 2015. If the values of $d(x)$ are written as a sequence in non-decreasing order, find the sum of the first 2015 numbers in the sequence.	1,015,056
2	Convert 731_8 to a base-4 number.	13121_4 or 13121
3	In how many points in the *xy*-plane do the polar graphs $r=\theta$ and $r=\cos\theta$ intersect?	2
4	A planar graph consists of 47 edges, each of which connects two each of 30 vertices. No two edges connect the same two vertices directly, and no two edges intersect. Into how many non-overlapping regions does this graph divide the plane?	19
5	Baby Björn has a total of \$20.09 to his name in the entire world, all in pennies, nickels, dimes, and quarters. Coincidentally, he has the same number of pennies as nickels as dimes as quarters. How many total coins does Baby Björn have in the entire world?	196

5 points each		
6	The function $f(x)=-3x^2-12x+14$ has a relative maximum at what point?	$(-2,26)$
7	Find the area, in square meters, enclosed by an ellipse with major and minor axes of lengths 18 meters and 12 meters, respectively.	54π
8	If you are dealt three cards from a standard deck of 52 playing cards, what is the probability that the three cards are of consecutive ranks? "Rank" refers to the value on the card: 2, 3, …, Q, K, A. For this problem, count the ace only as a high card—thus, the cards A, 2, and 3 would not be considered as three cards of consecutive ranks, but the cards Q, K, and A would.	$\frac{176}{5525}$
9	Find the product of the greatest common divisor and the least common multiple of 2015 and 1105.	2,226,575
10	Consider the 71 fractions $\frac{1}{72}, \frac{2}{72}, \ldots, \frac{71}{72}$. If each of these fractions is reduced into lowest terms, in how many of these lowest-terms fractions is the numerator equal to 1?	11

6 points each		
11	Define a sequence $\langle a_n \rangle_{n=1}^{\infty}$ as $a_1 = 1$ and for any integer $n \geq 2$, $a_n = \dfrac{\sum_{i=1}^{n-1} a_i}{\prod_{i=1}^{n-1} a_i}$. Find the numerical value of a_8.	$\dfrac{17}{15}$
12	Let $f(x) = 2x^2 - 1$. Find the values of x that satisfy $f(f(x)) = f(x)$.	$\pm 1,\ \pm\dfrac{1}{2}$ (all 4, any order)
13	Find the coefficient of x^5 in the expansion of $(x^3 + 2x^2 - 3)(x-1)(x+1)^2$.	3
14	From the endpoints of the latus rectum of the parabola with equation $y = \dfrac{x^2}{36} - \dfrac{x}{6} - \dfrac{3}{4}$, two line segments are drawn perpendicularly to the directrix of the parabola. The latus rectum, the two line segments, and a portion of the directrix form a rectangle that encloses what area?	648
15	A circle has a radius of length $2\sqrt{\cot 20^\circ}$ inches. Find the area, in square inches, enclosed by the regular nonagon circumscribed about the circle.	36

2014 – 2015 Log1 Contest Round 3
Mu Individual

Name: ________________

	4 points each	
1	For all integers x, let $d(x)$ be the distance on the number line between x and 2015. If the values of $d(x)$ are written as a sequence in non-decreasing order, find the sum of the first 2015 numbers in the sequence.	1,015,056
2	Consider the graph of $6x^2 - xy - 35y^2 = 0$. At each point on this graph except for the origin, the slope of a tangent line to that point must have one of two distinct slopes. Find the lesser of those two slopes.	$-\frac{3}{7}$
3	Evaluate: $\lim\limits_{x\to-\infty} \frac{3-x}{\sqrt{4x^2+6x+7}}$	$\frac{1}{2}$ or 0.5
4	A planar graph consists of 47 edges, each of which connects two each of 30 vertices. No two edges connect the same two vertices directly, and no two edges intersect. Into how many non-overlapping regions does this graph divide the plane?	19
5	If $f(x) = 4x^3 - \arcsin x$, find the value of $f'\left(\frac{\sqrt{3}}{2}\right)$.	7

	5 points each	
6	The function $f(x) = -2x^3 + 5x^2 - 4x - 1$ has a relative maximum at what point?	$(1,-2)$
7	If $\int_{-5}^{-4} \frac{3x+6}{x^2+3x}\,dx = \ln\left(\frac{a}{b}\right)$, where a and b are relatively prime positive integers, find the value of $b-a$.	17
8	If you are dealt three cards from a standard deck of 52 playing cards, what is the probability that the three cards are of consecutive ranks? "Rank" refers to the value on the card: 2, 3, …, Q, K, A. For this problem, count the ace only as a high card—thus, the cards A, 2, and 3 would not be considered as three cards of consecutive ranks, but the cards Q, K, and A would.	$\frac{176}{5525}$
9	Consider the 71 fractions $\frac{1}{72}, \frac{2}{72}, \ldots, \frac{71}{72}$. If each of these fractions is reduced into lowest terms, in how many of these lowest-terms fractions is the numerator equal to 1?	11
10	Define a sequence $\langle a_n \rangle_{n=1}^{\infty}$ as $a_1 = 1$ and for any integer $n \geq 2$, $a_n = \frac{\sum_{i=1}^{n-1} a_i}{\prod_{i=1}^{n-1} a_i}$. Find the numerical value of a_8.	$\frac{17}{15}$

6 points each		
11	Find the average of the positive integral factors of 2015.	336
12	Evaluate to the nearest 0.01: $\frac{1}{\sqrt{2\pi}}\int_{-1}^{1} e^{-0.5x^2}\,dx$	0.68
13	Find the coefficient of x^5 in the expansion of $\left(x^3+2x^2-3\right)\left(x-1\right)\left(x+1\right)^2$.	3
14	Consider the parabola with equation $y=\frac{x^2}{36}-\frac{x}{6}-\frac{3}{4}$. The tangents to this parabola at the endpoints of its latus rectum intersect at what point?	$\left(3,-10\right)$
15	A circle has a radius of length $\sqrt{2}+\sqrt{6}$ inches. Find the area, in square inches, enclosed by the regular dodecagon circumscribed about the circle.	48

2014 – 2015 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
1	1	1	2015 would have the least distance (0), followed by 2014 & 2016 (1), 2013 & 2017 (2), and so on. Therefore there is one integer whose distance is 0 and two integers each whose distance is each integral value from 1 to 1007. Therefore, the sum of the 2015 least values of $d(x)$ is $0+2\cdot\frac{1007\cdot1008}{2}=1{,}015{,}056$.
		2	$2^8=2^{2^3}=65536^{2^x}=\left(2^{16}\right)^{2^x}=2^{16\cdot2^x}\Rightarrow 8=16\cdot2^x\Rightarrow 2^x=\frac{1}{2}\Rightarrow x=-1$
	2	3	Since 8 and 4 are the 3rd and 2nd powers of 2, you can convert each two digits of the given number into a three-digit number for the base-4 number. $31_8=3\cdot8+1_{10}=25_{10}$ $=1\cdot16+2\cdot8+1_{10}=121_4$ and $7_8=7_{10}=1\cdot4+3\cdot1_{10}=13_4$, so the answer is 13121_4.
2			$0=6x^2-xy-35y^2=(3x+7y)(2x-5y)$, so the graph is the two lines $y=-\frac{3}{7}x$ and $y=\frac{2}{5}x$, making the slope of the tangent line at any point except the origin (the intersection of the two lines) $-\frac{3}{7}$ or $\frac{2}{5}$ — the least of these is $-\frac{3}{7}$.
	3		$r=\theta$ is a spiral emanating from the origin, and $r=\cos\theta$ is a circle centered at $(0.5,0)$ with radius of length 0.5, thus lying only in quadrants I and IV. Sketching these graphs, there are intersections at the origin and at only one point in the first quadrant (the second time the spiral comes around in quadrant I, the distance from the origin is already at least 2π, too far away for the graphs to intersect). To see that these graphs don't intersect in quadrant III, the closest points on the spiral are a distance at least $\frac{3\pi}{2}$ from the origin, much too far to intersect the given circle. Therefore, there are two intersections in the plane of these two graphs: the origin and some point in quadrant I.
3			The expression is defined as $x\to-\infty$, and the sign of the quantity will be positive since the numerator and denominator both are positive. Therefore, $\lim\limits_{x\to-\infty}\frac{3-x}{\sqrt{4x^2+6x+7}}=$ $\lim\limits_{x\to-\infty}\sqrt{\frac{x^2-6x+9}{4x^2+6x+7}}=\sqrt{\lim\limits_{x\to-\infty}\frac{x^2-6x+9}{4x^2+6x+7}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$.
4	4	4	This is simply Euler's formula, so $2=V-E+F=30-47+F\Rightarrow F=19$.
	5	5	One each of the coins adds up to 41 cents, so the total number of each coin would be $20.09\div0.41=49$, making the total number of all coins $49\cdot4=196$.
5			$f'(x)=12x^2-\frac{1}{\sqrt{1-x^2}}\Rightarrow f'\left(\frac{\sqrt{3}}{2}\right)=12\left(\frac{\sqrt{3}}{2}\right)^2-\frac{1}{\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}}=12\left(\frac{3}{4}\right)-\frac{1}{\sqrt{\frac{1}{4}}}=9-2=7$

		6	Suppose the angle bisector divides the side into pieces of length *x* and $28-x$, where the side with length *x* is closer to the side with length 26. Then $\frac{26}{x}=\frac{30}{28-x}\Rightarrow x=13$ and $28-x=15$. Then the angle bisector has length $\sqrt{30\cdot 26-15\cdot 13}=\sqrt{585}=3\sqrt{65}$.
	6		This is a parabola, opening downward, whose vertex occurs when $x=-\frac{-12}{2(-3)}=-2$. The *y*-coordinate at the vertex is $f(-2)=-3(-2)^2-12(-2)+14=26$, so the vertex, which is also the relative maximum, is $(-2,26)$.
6			This is a cubic polynomial with a negative leading coefficient, so the relative maximum occurs at the greater of the two critical numbers. $f'(x)=-6x^2+10x-4$ $=-2(3x-2)(x-1)$, so the relative maximum occurs when $x=1$. The *y*-coordinate of this point is $f(1)=-2(1)^3+5(1)^2-4(1)-1=-2$, so the relative maximum is at $(1,-2)$.
	7	7	The semi-major and semi-minor axes have lengths 9 and 6, respectively, so the enclosed area is $\pi\cdot 9\cdot 6=54\pi$.
7			$\int_{-5}^{-4}\frac{3x+6}{x^2+3x}dx=\int_{-5}^{-4}\frac{3x^2+6x}{x^3+3x^2}dx=\ln\left\|x^3+3x^2\right\|\Big\|_{-5}^{-4}=\ln\|-16\|-\ln\|-50\|=\ln\frac{16}{50}=\ln\frac{8}{25}$, so $b-a$ $=25-8=17$.
		8	You must choose 1 of the 4 suits, then 3 of the 13 cards in that suit. Further, you are choosing 3 cards from a total of 52 cards, so the probability of getting three cards of the same suit is $\frac{\binom{4}{1}\binom{13}{3}}{\binom{52}{3}}=\frac{\frac{4!}{1!3!}\cdot\frac{13!}{3!10!}}{\frac{52!}{3!49!}}=\frac{4\cdot 13\cdot 2\cdot 11}{26\cdot 17\cdot 50}=\frac{22}{425}$.
8	8		There are 11 possible ranks for lowest rank in the three card hand (2, 3, ..., Q), and knowing the lowest rank dictates the other two ranks. You must select 1 of the 4 cards within each rank, so the probability is $\frac{\binom{11}{1}\binom{4}{1}^3}{\binom{52}{3}}=\frac{11\cdot 4^3}{26\cdot 17\cdot 50}=\frac{176}{5525}$.
	9	9	The product of the greatest common factor and the least common multiple of two numbers equals the product of the two number themselves—thus, the answer is $2015\cdot 1105=2{,}226{,}575$. If you were unaware of this relationship, since 2015 $=5\cdot 13\cdot 31$ and $1105=5\cdot 13\cdot 17$, the greatest common factor is $5\cdot 13=65$ and the least common multiple is $5\cdot 13\cdot 17\cdot 31=34{,}255$, so this product is $65\cdot 34{,}255=2{,}226{,}575$.
		10	If *a* is the length of a side of the octagon, then the octagon's enclosed area is $\left(2+2\sqrt{2}\right)a^2$. If *b* is the length of a side of the hexagon, then the hexagon's enclosed area is $\frac{3\sqrt{3}}{2}b^2$. Therefore, $\left(2+2\sqrt{2}\right)a^2=\frac{3\sqrt{3}}{2}b^2\Rightarrow\frac{a^2}{b^2}=\frac{3\sqrt{3}}{4+4\sqrt{2}}=\frac{3\sqrt{6}-3\sqrt{3}}{4}$.

9	10	11	The fraction will have a 1 in the numerator if the numerator in the unreduced fraction is a factor of 72. Since $72=2^3\cdot 3^2$, 72 has a total of $(3+1)(2+1)=12$ positive integral factors. However, one of those factors, 72, is not a numerator of any of the fractions, so the number of fractions with a 1 in the numerator will be 11 (the numerators for which this works are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, and 36).
10	11	12	Since this is a recursive sequence, we must find the first eight terms in order to find the eighth term. $a_1=1$, $a_2=\frac{1}{1}=1$, $a_3=\frac{1+1}{1\cdot 1}=2$, $a_4=\frac{1+1+2}{1\cdot 1\cdot 2}=2$, $a_5=\frac{1+1+2+2}{1\cdot 1\cdot 2\cdot 2}=\frac{3}{2}$, $a_6=\frac{1+1+2+2+3/2}{1\cdot 1\cdot 2\cdot 2\cdot 3/2}=\frac{5}{4}$, $a_7=\frac{1+1+2+2+3/2+5/4}{1\cdot 1\cdot 2\cdot 2\cdot 3/2\cdot 5/4}=\frac{7}{6}$, and $a_8=\frac{1+1+2+2+3/2+5/4+7/6}{1\cdot 1\cdot 2\cdot 2\cdot 3/2\cdot 5/4\cdot 7/6}=\frac{17}{15}$, so the answer is $\frac{17}{15}$.
11			Since the prime factorization of 2015 is $5\cdot 13\cdot 31$, the sum of the positive integral factors of 2015 is $(1+5)(1+13)(1+31)=6\cdot 14\cdot 32=2688$ and the number of positive integral factors of 2015 is $(1+1)^3=2^3=8$. Therefore, the average of all positive integral factors of 2015 is $\frac{2688}{8}=336$.
	12		$f(f(x))=f(2x^2-1)=2(2x^2-1)^2-1=8x^4-8x^2+1$, so $8x^4-8x^2+1=2x^2-1\Rightarrow$ $0=8x^4-10x^2+2=2(4x^2-1)(x^2-1)\Rightarrow x=\pm\frac{1}{2}$ or $x=\pm 1$.
12			This corresponds to the area within one standard deviation of the mean under a standard normal curve, which rounded to the nearest 0.01 is 0.68.
13	13	13	Each term in the expansion is the product of one term from each of the factors, so the only ways to get an x^5 term are 1) the x^3 term in the cubic factor, two *x* terms in the linear factors, and a constant in the third linear factor; or 2) the x^2 term in the cubic factor and the *x* terms in all three linear factors. Therefore, the coefficient of x^5 in the expansion is $1\cdot 1\cdot 1\cdot 1+1\cdot 1\cdot 1\cdot 1+1\cdot 1\cdot 1\cdot -1+2\cdot 1\cdot 1\cdot 1=1+1-1+2=3$.
		14	The length of the latus rectum of a parabola is the reciprocal of the square term, so in this case, that length is 36.
	14		The length of the latus rectum of a parabola is the reciprocal of the square term, so in this case, that length is 36. The perpendicular distance from the latus rectum to the directrix is half the length of the latus rectum, so in this case, that length is 18. Therefore, the enclosed area of the rectangle is $36\cdot 18=648$.
14			The vertex form for this parabola is $y=\frac{1}{36}(x-3)^2-1$, so the vertex is at the point $(3,-1)$. The focus of the parabola is then at the point $(3,8)$, and since the latus rectum has length 36 (the reciprocal of the coefficient of the square term), the endpoints of the latus rectum are $(-15,8)$ and $(21,8)$. Since $y'=\frac{x-3}{18}$, $y'(-15)=-1$ and $y'(21)=1$, meaning that the tangents to those two points have equations $y=-x-7$ and $y=x-13$, respectively. Those two lines intersect at the point $(3,-10)$.

		15	The circle's radius is the apothem for the octagon, and the length of the apothem is $\frac{1}{2}s+\frac{\sqrt{2}}{2}s=\left(\frac{1+\sqrt{2}}{2}\right)s$, where s is the length of a side of the octagon. Therefore, $\frac{\sqrt{2}}{2}=\left(\frac{1+\sqrt{2}}{2}\right)s \Rightarrow s=2-\sqrt{2}$, making the area enclosed by the octagon $A=\frac{1}{2}a(8s)$ $=4\cdot\frac{\sqrt{2}}{2}\left(2-\sqrt{2}\right)=4\sqrt{2}-4$.
	15		The circle's radius is the apothem for the nonagon, and using right triangle trigonometry, the length of the side of the nonagon is $2r\tan 20^\circ$, where r is the length of the apothem. Therefore, the area enclosed by the nonagon is $A=\frac{1}{2}r\left(9\cdot 2r\tan 20^\circ\right)$ $=9r^2\tan 20^\circ=9\left(2\sqrt{\cot 20^\circ}\right)^2\tan 20^\circ=36\cot 20^\circ\tan 20^\circ=36$.
15			The circle's radius is the apothem for the dodecagon, and using right triangle trigonometry, the length of the side of the dodecagon is $2r\tan 15^\circ=\left(4-2\sqrt{3}\right)r$, where r is the length of the apothem. Therefore, the area enclosed by the dodecagon is $A=\frac{1}{2}r\left(12\cdot\left(4-2\sqrt{3}\right)r\right)=\left(24-12\sqrt{3}\right)r^2=\left(24-12\sqrt{3}\right)\left(\sqrt{2}+\sqrt{6}\right)^2=48$.