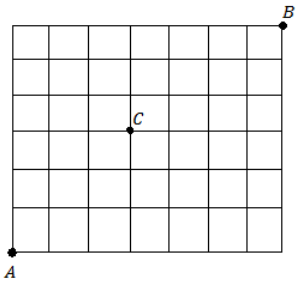
2014 – 2015 Log1 Contest Round 2
Theta Probability

Name: _____

4 points each		
1	What is the probability of rolling an even number when rolling a standard, six-sided die once?	
2	If I flip a fair coin three times, what is the probability that exactly two of the flips are the tails?	
3	If I flip a fair coin three times, what is the probability that exactly two of the flips are the same (either heads or tails)?	
4	If I roll two standard, 20-sided dice (sides numbered integers 1 through 20), what is the probability of rolling a sum of 5 or more?	
5	How many of the following ten numbers can be the probability of an event? $\frac{1}{e}$, $\frac{e}{\pi}$, $\frac{-1+\sqrt{5}}{2}$, $\cos 49^\circ$, $\log_3 2$, $\ln 3$, 0, $\log_{0.75} 8$, $\sec(2\pi)$, $10{,}000{,}000^{-0.5}$	

5 points each		
6	What is the probability of drawing from a standard, 52-card deck of playing cards a king, then a heart, then a non-face card, if drawn cards are put back in the deck once they are drawn?	
7	Ishant and Yuan play a game where they toss a fair coin 7 times. Ishant wins if 4 or more of the flips come up heads, with the exception that if all 7 flips come up heads (no one should have that much luck); otherwise, Yuan wins. What is the probability that Yuan wins?	
8	Consider the grid to the right. Daniel Tiger travels along the grid, starting at point *A* and ending at point *B*. He may only move along the lines of the grid, and he may only move up or to the right at any given time. What is the probability that among all such paths Daniel Tiger could take, a randomly chosen path must also pass through point *C*? 	
9	A fair coin is tossed six times. What is the probability that one side (heads or tails) comes up more than the other?	
10	A bag contains 20 red, 10 yellow, 2 blue, and 8 green marbles. If I choose one marble from the bag, what is the probability that it is a primary color?	

6 points each		
11	A piece of yarn is cut at a randomly selected point along its length, creating two pieces of yarn. What is the probability that the longer piece of yarn is at least 25% longer than the shorter piece?	
12	A coin is weighted so that tails comes up 40% of the time, heads comes up 40% of the time, and in 20% of the flips, the coin lands on its edge (it is a most peculiar coin indeed). If I flip this coin five times, what is the probability that tails comes up exactly 3 times?	
13	A pin code consists of four digits, each an integer from 0 to 9, inclusive. In order to be more secure, no two consecutive digits may be equal in number (so, for example, 88 may not show up anywhere in the pin code). How many such pin codes are possible?	
14	Consider the region consisting of all points whose distance from the point $(2,4)$ or distance from the point $(8,4)$ is less than or equal to 3. Find the probability that a point chosen from this region has both such distances less than or equal to 3.	
15	Consider the region consisting of all points whose distance from the point $(2,4)$ or distance from the point $(8,4)$ is less than or equal to 6. If the probability that a point chosen from this region has both such distances less than or equal to 6 is given in the form $\frac{a\pi - b\sqrt{3}}{c\pi + d\sqrt{3}}$, where *a*, *b*, *c*, and *d* are relatively prime positive integers, find the value of $a+b+c+d$.	

2014 – 2015 Log1 Contest Round 2
Alpha Probability

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5	What is the probability of drawing from a standard, 52-card deck of playing cards a king, then a heart, then a non-face card, if drawn cards are put back in the deck once they are drawn?	

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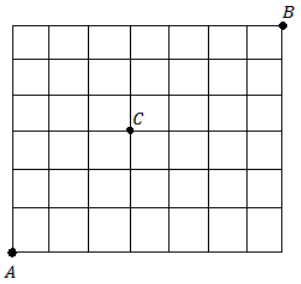
6	Ishant and Yuan play a game where they toss a fair coin 7 times. Ishant wins if 4 or more of the flips come up heads, with the exception that if all 7 flips come up heads (no one should have that much luck); otherwise, Yuan wins. What is the probability that Yuan wins?	
7	A triangle has two sides of lengths 6 and 9, and the third side of the triangle also has an integer length. What is the probability that the area enclosed by the triangle is an integer multiple of $\sqrt{2}$?	
8	Consider the grid to the right. Daniel Tiger travels along the grid, starting at point *A* and ending at point *B*. He may only move along the lines of the grid, and he may only move up or to the right at any given time. What is the probability that among all such paths Daniel Tiger could take, a randomly chosen path must also pass through point *C*? 	
9	A fair coin is tossed six times. What is the probability that one side (heads or tails) comes up more than the other?	
10	A coin is weighted so that tails comes up 60% of the time while heads comes up the other 40%. If I flip this coin five times, what is the probability that tails comes up exactly 3 times?	

6 points each		
11	A piece of yarn is cut at a randomly selected point along its length, creating two pieces of yarn. What is the probability that the longer piece of yarn is at least 25% longer than the shorter piece?	
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13	A pin code consists of four digits, each an integer from 0 to 9, inclusive. In order to be more secure, all four digits must be distinct, but the pin code may not consist of strictly increasing or strictly decreasing numbers when reading the pin code from left to right (so, for example, neither 1348 nor 8431 is an acceptable pin code, but 1384 is acceptable, as is 8134). How many such pin codes are possible?	
14	Consider the region consisting of all points whose distance from the point $(2,4)$ or distance from the point $(8,4)$ is less than or equal to 6. If the probability that a point chosen from this region has both such distances less than or equal to 6 is given in the form $\frac{a\pi - b\sqrt{3}}{c\pi + d\sqrt{3}}$, where *a*, *b*, *c*, and *d* are relatively prime positive integers, find the value of $a+b+c+d$.	
15	Let *p* be the probability of a possible event. For what values of *p* does the infinite exponentiation $p^{p^{p^{p^{\cdot^{\cdot^{\cdot}}}}}}$ converge? Write your answer in interval notation.	

2014 – 2015 Log1 Contest Round 2
Mu Probability

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4 points each		
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2	If I flip a fair coin three times, what is the probability that exactly two of the flips are the tails?	
3	If I flip a fair coin three times, what is the probability that exactly two of the flips are the same (either heads or tails)?	
4	If I roll two standard, 20-sided dice (sides numbered integers 1 through 20), what is the probability of rolling a sum of 20 or more?	
5	What is the probability of drawing from a standard, 52-card deck of playing cards a king, then a heart, then a non-face card, if drawn cards are put back in the deck once they are drawn?	

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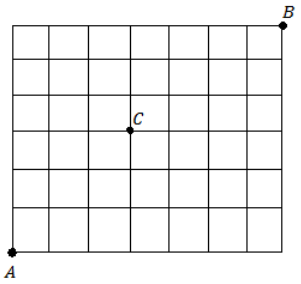
6	Ishant and Yuan play a game where they toss a fair coin 7 times. Ishant wins if 4 or more of the flips come up heads, with the exception that if all 7 flips come up heads (no one should have that much luck); otherwise, Yuan wins. What are the odds that Yuan wins?	
7	A triangle has two sides of lengths 6 and 9, and the third side of the triangle also has an integer length. What is the probability that the area enclosed by the triangle is an integer multiple of $\sqrt{2}$?	
8	Consider the grid to the right. Daniel Tiger travels along the grid, starting at point *A* and ending at point *B*. He may only move along the lines of the grid, and he may only move up or to the right at any given time. What is the probability that among all such paths Daniel Tiger could take, a randomly chosen path must also pass through point *C*? 	
9	A coin is weighted so that tails comes up 60% of the time while heads comes up the other 40%. If I flip this coin five times, what is the probability that tails comes up exactly 3 times?	
10	A piece of yarn is cut at a randomly selected point along its length, creating two pieces of yarn. What is the probability that the longer piece of yarn is at least 25% longer than the shorter piece?	

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12	A bag contains four coins. One coin is fair, and for the other three coins, the probability of flipping a heads is $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{5}{6}$, respectively. If you draw one coin from the bag, each coin being equally likely to be selected, and end up flipping heads on it, what is the probability that the coin you flipped was the fair coin?	
13	Consider the region consisting of all points whose distance from the point $(2,4)$ or distance from the point $(8,4)$ is less than or equal to 6. If the probability that a point chosen from this region has both such distances less than or equal to 6 is given in the form $\frac{a\pi - b\sqrt{3}}{c\pi + d\sqrt{3}}$, where *a*, *b*, *c*, and *d* are relatively prime positive integers, find the value of $a+b+c+d$.	
14	Marshmallow rolls two standard, six-sided dice, and he multiplies the rolled faces together. What is the probability that this product is 16 or higher?	
15	Let *p* be the probability of a possible event. For what values of *p* does the infinite exponentiation $p^{p^{p^{p^{\cdot^{\cdot^{\cdot}}}}}}$ converge? Write your answer in interval notation.	

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Theta Probability

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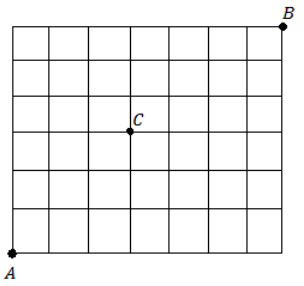
1	What is the probability of rolling an even number when rolling a standard, six-sided die once?	$\frac{1}{2}$ or 0.5
2	If I flip a fair coin three times, what is the probability that exactly two of the flips are the tails?	$\frac{3}{8}$ or 0.375
3	If I flip a fair coin three times, what is the probability that exactly two of the flips are the same (either heads or tails)?	$\frac{3}{4}$ or 0.75
4	If I roll two standard, 20-sided dice (sides numbered integers 1 through 20), what is the probability of rolling a sum of 5 or more?	$\frac{197}{200}$ or 0.985
5	How many of the following ten numbers can be the probability of an event? $\frac{1}{e}$, $\frac{e}{\pi}$, $\frac{-1+\sqrt{5}}{2}$, $\cos 49^\circ$, $\log_3 2$, $\ln 3$, 0, $\log_{0.75} 8$, $\sec(2\pi)$, $10{,}000{,}000^{-0.5}$	8

5 points each		
6	What is the probability of drawing from a standard, 52-card deck of playing cards a king, then a heart, then a non-face card, if drawn cards are put back in the deck once they are drawn?	$\frac{5}{338}$
7	Ishant and Yuan play a game where they toss a fair coin 7 times. Ishant wins if 4 or more of the flips come up heads, with the exception that if all 7 flips come up heads (no one should have that much luck); otherwise, Yuan wins. What is the probability that Yuan wins?	$\frac{65}{128}$
8	Consider the grid to the right. Daniel Tiger travels along the grid, starting at point *A* and ending at point *B*. He may only move along the lines of the grid, and he may only move up or to the right at any given time. What is the probability that among all such paths Daniel Tiger could take, a randomly chosen path must also pass through point *C*? 	$\frac{175}{429}$
9	A fair coin is tossed six times. What is the probability that one side (heads or tails) comes up more than the other?	$\frac{11}{16}$ or 0.6875
10	A bag contains 20 red, 10 yellow, 2 blue, and 8 green marbles. If I choose one marble from the bag, what is the probability that it is a primary color?	$\frac{4}{5}$ or 0.8

6 points each		
11	A piece of yarn is cut at a randomly selected point along its length, creating two pieces of yarn. What is the probability that the longer piece of yarn is at least 25% longer than the shorter piece?	$\frac{8}{9}$
12	A coin is weighted so that tails comes up 40% of the time, heads comes up 40% of the time, and in 20% of the flips, the coin lands on its edge (it is a most peculiar coin indeed). If I flip this coin five times, what is the probability that tails comes up exactly 3 times?	$\frac{144}{625}$ or 0.2304
13	A pin code consists of four digits, each an integer from 0 to 9, inclusive. In order to be more secure, no two consecutive digits may be equal in number (so, for example, 88 may not show up anywhere in the pin code). How many such pin codes are possible?	7290
14	Consider the region consisting of all points whose distance from the point $(2,4)$ or distance from the point $(8,4)$ is less than or equal to 3. Find the probability that a point chosen from this region has both such distances less than or equal to 3.	0
15	Consider the region consisting of all points whose distance from the point $(2,4)$ or distance from the point $(8,4)$ is less than or equal to 6. If the probability that a point chosen from this region has both such distances less than or equal to 6 is given in the form $\frac{a\pi - b\sqrt{3}}{c\pi + d\sqrt{3}}$, where *a*, *b*, *c*, and *d* are relatively prime positive integers, find the value of $a+b+c+d$.	18

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3	If I flip a fair coin three times, what is the probability that exactly two of the flips are the same (either heads or tails)?	$\frac{3}{4}$ or 0.75
4	If I roll two standard, 20-sided dice (sides numbered integers 1 through 20), what is the probability of rolling a sum of 10 or more?	$\frac{91}{100}$ or 0.91
5	What is the probability of drawing from a standard, 52-card deck of playing cards a king, then a heart, then a non-face card, if drawn cards are put back in the deck once they are drawn?	$\frac{5}{338}$

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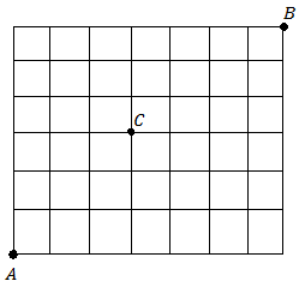
7	A triangle has two sides of lengths 6 and 9, and the third side of the triangle also has an integer length. What is the probability that the area enclosed by the triangle is an integer multiple of $\sqrt{2}$?	$\frac{2}{11}$
8	Consider the grid to the right. Daniel Tiger travels along the grid, starting at point *A* and ending at point *B*. He may only move along the lines of the grid, and he may only move up or to the right at any given time. What is the probability that among all such paths Daniel Tiger could take, a randomly chosen path must also pass through point *C*? 	$\frac{175}{429}$
9	A fair coin is tossed six times. What is the probability that one side (heads or tails) comes up more than the other?	$\frac{11}{16}$ or 0.6875
10	A coin is weighted so that tails comes up 60% of the time while heads comes up the other 40%. If I flip this coin five times, what is the probability that tails comes up exactly 3 times?	$\frac{216}{625}$ or 0.3456

6 points each		
11	A piece of yarn is cut at a randomly selected point along its length, creating two pieces of yarn. What is the probability that the longer piece of yarn is at least 25% longer than the shorter piece?	$\frac{8}{9}$
12	A coin is weighted so that tails comes up 40% of the time, heads comes up 40% of the time, and in 20% of the flips, the coin lands on its edge (it is a most peculiar coin indeed). If I flip this coin five times, what is the probability that tails comes up exactly 3 times?	$\frac{144}{625}$ or 0.2304
13	A pin code consists of four digits, each an integer from 0 to 9, inclusive. In order to be more secure, all four digits must be distinct, but the pin code may not consist of strictly increasing or strictly decreasing numbers when reading the pin code from left to right (so, for example, neither 1348 nor 8431 is an acceptable pin code, but 1384 is acceptable, as is 8134). How many such pin codes are possible?	4620
14	Consider the region consisting of all points whose distance from the point $(2,4)$ or distance from the point $(8,4)$ is less than or equal to 6. If the probability that a point chosen from this region has both such distances less than or equal to 6 is given in the form $\frac{a\pi - b\sqrt{3}}{c\pi + d\sqrt{3}}$, where *a*, *b*, *c*, and *d* are relatively prime positive integers, find the value of $a+b+c+d$.	18
15	Let *p* be the probability of a possible event. For what values of *p* does the infinite exponentiation $p^{p^{p^{p^{p^{\cdot^{\cdot^{\cdot}}}}}}}$ converge? Write your answer in interval notation.	$(0,1]$

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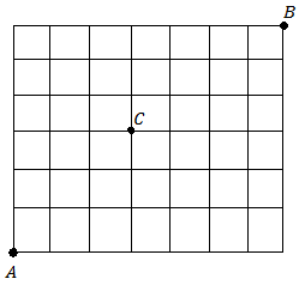
3	If I flip a fair coin three times, what is the probability that exactly two of the flips are the same (either heads or tails)?	$\frac{3}{4}$ or 0.75
4	If I roll two standard, 20-sided dice (sides numbered integers 1 through 20), what is the probability of rolling a sum of 20 or more?	$\frac{229}{400}$ or 0.5725
5	What is the probability of drawing from a standard, 52-card deck of playing cards a king, then a heart, then a non-face card, if drawn cards are put back in the deck once they are drawn?	$\frac{5}{338}$

5 points each		
6	Ishant and Yuan play a game where they toss a fair coin 7 times. Ishant wins if 4 or more of the flips come up heads, with the exception that if all 7 flips come up heads (no one should have that much luck); otherwise, Yuan wins. What are the odds that Yuan wins?	$\frac{65}{63}$ or 65:63
7	A triangle has two sides of lengths 6 and 9, and the third side of the triangle also has an integer length. What is the probability that the area enclosed by the triangle is an integer multiple of $\sqrt{2}$?	$\frac{2}{11}$
8	Consider the grid to the right. Daniel Tiger travels along the grid, starting at point *A* and ending at point *B*. He may only move along the lines of the grid, and he may only move up or to the right at any given time. What is the probability that among all such paths Daniel Tiger could take, a randomly chosen path must also pass through point *C*? 	$\frac{175}{429}$
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12	A bag contains four coins. One coin is fair, and for the other three coins, the probability of flipping a heads is $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{5}{6}$, respectively. If you draw one coin from the bag, each coin being equally likely to be selected, and end up flipping heads on it, what is the probability that the coin you flipped was the fair coin?	$\frac{6}{23}$
13	Consider the region consisting of all points whose distance from the point $(2,4)$ or distance from the point $(8,4)$ is less than or equal to 6. If the probability that a point chosen from this region has both such distances less than or equal to 6 is given in the form $\frac{a\pi - b\sqrt{3}}{c\pi + d\sqrt{3}}$, where *a*, *b*, *c*, and *d* are relatively prime positive integers, find the value of $a+b+c+d$.	18
14	Marshmallow rolls two standard, six-sided dice, and he multiplies the rolled faces together. What is the probability that this product is 16 or higher?	$\frac{11}{36}$
15	Let *p* be the probability of a possible event. For what values of *p* does the infinite exponentiation $p^{p^{p^{p^{\cdot^{\cdot^{\cdot}}}}}}$ converge? Write your answer in interval notation.	$(0,1]$

2014 – 2015 Log1 Contest Round 2
Probability Solutions

Mu	Al	Th	Solution
1	1	1	Sides 2, 4, and 6 are even, while sides 1, 3, and 5 are odd, so the probability is $\frac{3}{6}=\frac{1}{2}$.
2	2	2	Since there are two options for each flip, there are a total of $2^3=8$ possible sequences. There are $\binom{3}{2}=3$ ways to choose the two positions for the tails, so the probability of flipping exactly 2 tails is $\frac{3}{8}$.
3	3	3	This is similar to the last question, except that there are also $\binom{3}{2}=3$ ways to choose the two positions for heads, so the probability of flipping exactly two of the same thing is $\frac{3+3}{8}=\frac{6}{8}=\frac{3}{4}$.
		4	Count the number of ways of rolling a sum of 4 or less, since there are fewer of those. The combinations of rolls are 1 and 1 (1 way), 1 and 2 (2 ways), 1 and 3 (2 ways), and 2 and 2 (1 way), making a total of six possible rolls with a sum of 4 or less. The total number of rolls is $20^2=400$, so the probability is $1-\frac{6}{400}=\frac{394}{400}=\frac{197}{200}$.
	4		Count the number of ways of rolling a sum of 9 or less, since there are fewer of those. The number of combinations of rolls with a sum of 2 is 1, with a sum of 3 is 2, with a sum of 4 is 3, etc., on up to a sum of 9 is 8, making a total of $1+2+...+8=36$ possible rolls with a sum of 9 or less. The total number of rolls is $20^2=400$, so the probability is $1-\frac{36}{400}=\frac{364}{400}=\frac{91}{100}$.
4			Count the number of ways of rolling a sum of 19 or less, since there are fewer (though now many) of those. The number of combinations of rolls with a sum of 2 is 1, with a sum of 3 is 2, with a sum of 4 is 3, etc., on up to a sum of 19 is 18, making a total of $1+2+...+18=171$ possible rolls with a sum of 19 or less. The total number of rolls is $20^2=400$, so the probability is $1-\frac{171}{400}=\frac{229}{400}$.
		5	The probability of an event must be a number from 0 to 1, inclusive. Of the numbers listed, only $\ln 3$ (which is greater than 1) and $\log_{0.75}8$ (which is negative) are outside this range. Therefore, there are 8 possible probabilities among the numbers given.
5	5	6	Since drawn cards are replaced, multiply each of the three individual probabilities together. The three probabilities are $\frac{1}{13}$, $\frac{1}{4}$, and $\frac{10}{13}$ (non-face cards are non-Jacks, non-Queens, and non-Kings), respectively, so the probability is $\frac{1}{13}\cdot\frac{1}{4}\cdot\frac{10}{13}=\frac{5}{338}$.

	6	7	Yuan wins if the number of heads on the seven flips is 0, 1, 2, 3, or 7. Therefore, since there are $2^7 = 128$ equally likely sequences of flips, the probability that Yuan wins is $\frac{\binom{7}{0}+\binom{7}{1}+\binom{7}{2}+\binom{7}{3}+\binom{7}{7}}{128} = \frac{1+7+21+35+1}{128} = \frac{65}{128}$.
6			Yuan wins if the number of heads on the seven flips is 0, 1, 2, 3, or 7. Therefore, since there are $2^7 = 128$ equally likely sequences of flips, the probability that Yuan wins is $\frac{\binom{7}{0}+\binom{7}{1}+\binom{7}{2}+\binom{7}{3}+\binom{7}{7}}{128} = \frac{1+7+21+35+1}{128} = \frac{65}{128}$. This means Ishant's probability of winning is $1-\frac{65}{128} = \frac{63}{128}$, and the odds that Yuan wins would be $\frac{65/128}{63/128} = \frac{65}{63}$.
7	7		By the triangle inequality, the third side must be some integer from 4 to 14, inclusive. Let *x* be the length of the third side. By Hero's formula, the area enclosed by the triangle is $\sqrt{\frac{15+x}{2}\cdot\frac{15-x}{2}\cdot\frac{3+x}{2}\cdot\frac{x-3}{2}} = \frac{1}{4}\sqrt{\left(225-x^2\right)\left(x^2-9\right)}$. If *x* was even, then both factors under the radical would be odd, and there would be no factors of 2 that would cancel the $\frac{1}{4}$ in front of the radical—thus, *x* can't be even. In checking the five possible odd values of *x*, we need only check to see if $\left(225-x^2\right)\left(x^2-9\right)$ is a perfect square multiple of 32 (so that the square root of that quantity is an integer multiple of $4\sqrt{2}$).

x	$\left(225-x^2\right)\left(x^2-9\right)$	Perfect square multiple of 32?
5	$200\cdot 16 = 100\cdot 32$	Yes
7	$176\cdot 40 = 220\cdot 32$	No
9	$144\cdot 72 = 324\cdot 32$	Yes
11	$104\cdot 112 = 364\cdot 32$	No
13	$56\cdot 160 = 280\cdot 32$	No

			Therefore, 2 of the 11 possible values of *x* fit the condition, so the probability is $\frac{2}{11}$.
8	8	8	To go from *A* to *B*, any path constitutes 6 upward units and 7 rightward units. Once the rightward units are selected, the 6 upward units area already dictated, so the total number of paths from *A* to *B* is $\binom{13}{7} = 1716$. In a similar manner, the number of paths from *A* to *C* is $\binom{6}{3} = 20$, and the number of paths from *C* to *B* is $\binom{7}{4} = 35$. Therefore, the number of path from *A* to *B* passing through *C* is $20\cdot 35 = 700$. Therefore, the probability of passing through *C* is $\frac{700}{1716} = \frac{175}{429}$.
	9	9	The only way this would not happen would be if the same number of heads and tails came up, which would be 3 apiece. Thus, the probability is $1-\frac{\binom{6}{3}}{2^6} = 1-\frac{20}{64} = \frac{44}{64} = \frac{11}{16}$.

		10	The primary colors are red, yellow, and blue, so the total number of marbles that are primary colors is $\frac{20+10+2}{20+10+2+8}=\frac{32}{40}=\frac{4}{5}$.
9	10		$\binom{5}{3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2=\frac{10\cdot 3^3\cdot 2^2}{5^5}=\frac{216}{625}$
10	11	11	If the cut is made so that the two pieces are in the ratio of $5:4$, then this will be the break-even point. Anything closer to the end of the shorter side will work, so the right- or left-most $\frac{4}{9}$ of the yarn would work, giving a probability of $\frac{8}{9}$.
11	12	12	If we want exactly three tails, then heads or landing on edge can be lumped together as we don't care how many of those we get. Therefore, the probability is $\binom{5}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^2=\frac{10\cdot 2^3\cdot 3^2}{5^5}=\frac{144}{625}$.
12			Each coin has a $\frac{1}{4}$ probability of being selected, so using Bayes' theorem, the probability that the drawn coin was the fair one is $\frac{\frac{1}{4}\cdot\frac{1}{2}}{\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{5}{6}}=\frac{6}{23}$.
		13	There are 10 possible numbers for the first digit, but once that digit is selected, there are only 9 possible numbers for the second digit. After that, there are only 9 possible numbers for the third digit (any number except that used for the second digit) and 9 possible numbers for the fourth digit (any number except that used for the third digit). Therefore, there are $10\cdot 9^3=7290$ possible pin codes without the same digit used as consecutive digits.
	13		The number of ways to select four distinct digits is $\binom{10}{4}=210$, and then once you have four digits, there are $4!=24$ ways to order them. However, we must throw out two orderings for each set of four distinct numbers, since in one way they are strictly increasing and in another way they are strictly decreasing. Therefore, the number of possible pin codes that satisfy the condition is $210(24-2)=210\cdot 22=4620$.
		14	The only point that satisfies distance 3 or less from both given points is $(5,4)$, but there are an infinite amount of points from which to choose, so the probability is 0.
13	14	15	The region consists of two circular disks whose centers are on the boundary of each other. Therefore, we are looking for the area of the overlap divided by the total area. The overlap area consists of two equilateral triangular regions with side length 6 and four "pie crusts" (a 60° circular sector with the same equilateral triangular region removed). Therefore, the area of the overlap is $2\left(\frac{6^2\sqrt{3}}{4}\right)+4\left(\frac{1}{6}\pi\cdot 6^2-\frac{6^2\sqrt{3}}{4}\right)=24\pi-18\sqrt{3}$. The total area of the region is $2\pi\cdot(6^2)-(24\pi-18\sqrt{3})=48\pi+18\sqrt{3}$, so the probability is $\frac{24\pi-18\sqrt{3}}{48\pi+18\sqrt{3}}=\frac{4\pi-3\sqrt{3}}{8\pi+3\sqrt{3}}$, so $a+b+c+d=4+3+8+3=18$.

14			The possible rolls with a product of 16 or higher are 3-6 (2 ways), 4-4 (1 way), 4-5 (2 ways), 4-6 (2 ways), 5-5 (1 way), 5-6 (2 ways), and 6-6 (1 way), so because there are 36 possible rolls, the probability is $\frac{2+1+2+2+1+2+1}{36}=\frac{11}{36}$.
15	15		Set $x=p^{p^{p^{p^{\cdot^{\cdot^{\cdot}}}}}}$, then $x=p^x$, where $0<p\le 1$ (since *p* was the probability of a <u>possible</u> event). First, if $p=1$, then $x=1$ also, so the exponentiation converges. Now, consider $0<p<1$. Since $x=p^x$, the graphs of $y=x$ and $y=p^x$, $0<p<1$ are always going to intersect somewhere in the region $0<x<1, 0<y<1$. Therefore, the exponentiation will converge for all *p* in the interval $(0,1]$.