2014 – 2015 Log1 Contest Round 2
Theta Logarithms & Exponents

Name: ________________

4 points each		
1	Simplify: $\dfrac{(\log 243)(\log 625)(\log 216)}{(\log 36)(\log 729)(\log 25)}$	
2	Evaluate: $\log_2\left(2^{3^4}\cdot 4^{3^2}\right)$	
3	Find the sum: $1+1^1+1^{1^1}+1^{1^{1^1}}+\ldots+1^{1^{1^{1^{1^{1^{1^{1}}}}}}}$	
4	A square has a diagonal whose length is $\dfrac{\log_7\left(343^{\sqrt{2}}\right)}{\log_{81}\left(27^4\right)}$ feet. Find the area, in square feet, enclosed by the square.	
5	Evaluate: $(\log_8 544)(\log_{23} 716)(\log_7 219)(\log_3 1)$	

5 points each		
6	Given that $f(x)=4\cdot 2^{x+2}$, find the value of $f^{-1}(128)$.	
7	Simplify: $\log_3\left(9^{e^{\ln\left(e^{\ln 5}\right)}}\right)$	
8	If $2^{3x+1}=4^{2a}$ and $7^{6x+3}=\dfrac{1}{7^{5a}}$, find the numerical value of $\dfrac{x}{a}$.	
9	The number of bacteria in a culture grows exponentially according to the equation $\ln\left(\dfrac{N}{N_0}\right)=0.05t$, where N_0 is the initial population of the bacteria culture and N is the population of the culture after t hours. To the nearest hour, how long does it take for the culture to double its initial population?	
10	A bank offers an interest-bearing savings account at 4% annual interest, compounded semiannually. If I opened this type of account at this bank with an initial deposit of \$100, and if I make no deposits or withdrawals from the account, how much would I have in my account after two years (rounded down to the nearest cent)?	

6 points each		
11	Solve for x: $\log 9+\log 16=\log_x 12$	
12	Find the sum of all complex roots of the equation : $9^{7x^3+3x^2+27x+3}=81^{5x^3-3x^2+17x+11}$	
13	Is the real solution to the equation $4^x+2^x=8$ greater than (>), less than (<), or equal to (=) 1? Write as your answer ">", "<", or "=".	
14	If the real solution to the equation $36^x-6\left(24^x\right)-16^{x+1}=0$ can be written in the form $x=\log_y 8$, find the value of y.	
15	Find the sum of the common logarithms of all positive integral factors of 100,000.	

2014 – 2015 Log1 Contest Round 2
Alpha Logarithms & Exponents

Name: ________________

4 points each		
1	Simplify: $\dfrac{(\log 243)(\log 625)(\log 216)}{(\log 36)(\log 729)(\log 25)}$	
2	Evaluate: $\log_2\left(2^{3^4}\cdot 4^{3^2}\right)$	
3	Solve for the greater value of x: $2^{8x-x^2}=\sqrt{128\sqrt{256\sqrt{16}}}$	
4	A square has a diagonal whose length is $\dfrac{\log_7\left(343^{\sqrt{2}}\right)}{\log_{81}\left(27^4\right)}$ feet. Find the area, in square feet, enclosed by the square.	
5	Evaluate: $\left\lvert e^{5\pi i}\right\rvert$	

5 points each		
6	Given that $f(x)=4\cdot 2^{x+2}$, find the value of $f^{-1}(128)$.	
7	Given that $\log 3=a$, $\log 5=b$, and $\log 7=c$, find $(\log_{21}15)(\log_3 7)(\log_{49}5)(\log_5 9)$ in terms of a, b, and c.	
8	If $2^{3x+1}=4^{2a}$ and $7^{6x+3}=\dfrac{1}{7^{5a}}$, find the numerical value of $\dfrac{x}{a}$.	
9	The number of bacteria in a culture grows exponentially with growth constant 0.05/hour, where the exponential function has base e. To the nearest hour, how long does it take for the culture to double its initial population?	
10	A bank offers an interest-bearing savings account at 4% annual interest, compounded semiannually. If I opened this type of account at this bank with an initial deposit of $100, and if I make no deposits or withdrawals from the account, how much would I have in my account after two years (rounded down to the nearest cent)?	

6 points each		
11	Solve for *x*: $\log 9+\log 16=\log_x 12$	
12	Find the sum of all real roots of the equation : $9^{7x^3+3x^2+27x+3}=81^{5x^3-3x^2+17x+11}$	
13	Find all real solutions to the equation: $4^x+2^x=8$	
14	If the real solution to the equation $36^x-6\left(24^x\right)-16^{x+1}=0$ can be written in the form $x=\log_y 8$, find the value of y.	
15	Find the sum of the common logarithms of all positive integral factors of 100,000.	

2014 – 2015 Log1 Contest Round 2
Mu Logarithms & Exponents

Name: ________________

4 points each		
1	Simplify: $\dfrac{(\log 243)(\log 625)(\log 216)}{(\log 36)(\log 729)(\log 25)}$	
2	Solve for the greater value of *x*: $2^{8x-x^2}=\sqrt{128\sqrt{256\sqrt{16}}}$	
3	Find the slope of the tangent to the graph of $y=(\log_2 e)(2^x)$ at the point whose *y*-coordinate is $\log_2\sqrt{e}$.	
4	Evaluate: $\left\|e^{5\pi i}\right\|$	
5	Given that $f(x)=4\cdot 2^{x+2}$, find the value of $f^{-1}(128)$.	

5 points each		
6	Find the slope of the tangent to the graph of $y=e^{\sin x}+\log_{\pi/2} x$ at the point $\left(\dfrac{\pi}{2},e+1\right)$.	
7	Given that $\log 3=a$, $\log 5=b$, and $\log 7=c$, find $(\log_{21}15)(\log_3 7)(\log_{49}5)(\log_5 9)$ in terms of a, b, and c.	
8	If $2^{3x+1}=4^{2a}$ and $7^{6x+3}=\dfrac{1}{7^{5a}}$, find the numerical value of $\dfrac{x}{a}$.	
9	The number of bacteria in a culture grows exponentially with growth constant 0.05/hour, where the exponential function has base *e*. To the nearest hour, how long does it take for the culture to double its initial population?	
10	A bank offers an interest-bearing savings account at 4% annual interest, compounded either annually or semiannually. I opened two accounts, one with each type of compounding. I made an initial deposit of $100 into each account, and I made no deposits to or withdrawals from either account for two years. If the amount of money in each account is rounded down to the nearest cent, then how much more money would I have in the semiannually compounded account after those two years?	

6 points each		
11	Solve for x: $\log 9+\log 16=\log_x 12$	
12	Find the sum of all imaginary roots of the equation : $9^{7x^3+3x^2+27x+3}=81^{5x^3-3x^2+17x+11}$	
13	Find the sum of the common logarithms of all positive integral factors of 100,000.	
14	Evaluate: $\int_0^{\frac{\pi}{2}}\cos^4\theta d\theta$	
15	If the area enclosed by the graphs of $y=2^x$, $y=\log_2 x$, $x=1$, and $x=2$ can be written in the form $\log_2 y$, find the value of y.	

2014 – 2015 Log1 Contest Round 2
Theta Logarithms & Exponents

Name: ________________

4 points each		
1	Simplify: $\dfrac{(\log 243)(\log 625)(\log 216)}{(\log 36)(\log 729)(\log 25)}$	$\dfrac{5}{2}$ or 2.5
2	Evaluate: $\log_2\left(2^{3^4}\cdot 4^{3^2}\right)$	99
3	Find the sum: $1+1^1+1^{1^1}+1^{1^{1^1}}+\ldots+1^{1^{1^{1^{1^{1^{1^{1}}}}}}}$	8
4	A square has a diagonal whose length is $\dfrac{\log_7\left(343^{\sqrt{2}}\right)}{\log_{81}\left(27^4\right)}$ feet. Find the area, in square feet, enclosed by the square.	1
5	Evaluate: $(\log_8 544)(\log_{23} 716)(\log_7 219)(\log_3 1)$	0

5 points each		
6	Given that $f(x)=4\cdot 2^{x+2}$, find the value of $f^{-1}(128)$.	3
7	Simplify: $\log_3\left(9^{e^{\ln\left(e^{\ln 5}\right)}}\right)$	10
8	If $2^{3x+1}=4^{2a}$ and $7^{6x+3}=\dfrac{1}{7^{5a}}$, find the numerical value of $\dfrac{x}{a}$.	$\dfrac{17}{3}$
9	The number of bacteria in a culture grows exponentially according to the equation $\ln\left(\dfrac{N}{N_0}\right)=0.05t$, where N_0 is the initial population of the bacteria culture and N is the population of the culture after t hours. To the nearest hour, how long does it take for the culture to double its initial population?	14
10	A bank offers an interest-bearing savings account at 4% annual interest, compounded semiannually. If I opened this type of account at this bank with an initial deposit of $100, and if I make no deposits or withdrawals from the account, how much would I have in my account after two years (rounded down to the nearest cent)?	$108.24

6 points each		
11	Solve for x: $\log 9+\log 16=\log_x 12$	$\sqrt{10}$
12	Find the sum of all complex roots of the equation : $9^{7x^3+3x^2+27x+3}=81^{5x^3-3x^2+17x+11}$	3
13	Is the real solution to the equation $4^x+2^x=8$ greater than (>), less than (<), or equal to (=) 1? Write as your answer ">", "<", or "=".	>
14	If the real solution to the equation $36^x-6\left(24^x\right)-16^{x+1}=0$ can be written in the form $x=\log_y 8$, find the value of y.	$\frac{3}{2}$ or 1.5
15	Find the sum of the common logarithms of all positive integral factors of 100,000.	90

2014 – 2015 Log1 Contest Round 2
Alpha Logarithms & Exponents

Name: ________________

	4 points each	
1	Simplify: $\dfrac{(\log 243)(\log 625)(\log 216)}{(\log 36)(\log 729)(\log 25)}$	$\dfrac{5}{2}$ or 2.5
2	Evaluate: $\log_2\left(2^{3^4}\cdot 4^{3^2}\right)$	99
3	Solve for the greater value of x: $2^{8x-x^2}=\sqrt{128\sqrt{256\sqrt{16}}}$	$4+\sqrt{10}$
4	A square has a diagonal whose length is $\dfrac{\log_7\left(343^{\sqrt{2}}\right)}{\log_{81}\left(27^4\right)}$ feet. Find the area, in square feet, enclosed by the square.	1
5	Evaluate: $\left\lvert e^{5\pi i}\right\rvert$	1

	5 points each	
6	Given that $f(x)=4\cdot 2^{x+2}$, find the value of $f^{-1}(128)$.	3
7	Given that $\log 3=a$, $\log 5=b$, and $\log 7=c$, find $(\log_{21}15)(\log_3 7)(\log_{49}5)(\log_5 9)$ in terms of a, b, and c.	$\dfrac{a+b}{a+c}$
8	If $2^{3x+1}=4^{2a}$ and $7^{6x+3}=\dfrac{1}{7^{5a}}$, find the numerical value of $\dfrac{x}{a}$.	$\dfrac{17}{3}$
9	The number of bacteria in a culture grows exponentially with growth constant 0.05/hour, where the exponential function has base *e*. To the nearest hour, how long does it take for the culture to double its initial population?	14
10	A bank offers an interest-bearing savings account at 4% annual interest, compounded semiannually. If I opened this type of account at this bank with an initial deposit of $100, and if I make no deposits or withdrawals from the account, how much would I have in my account after two years (rounded down to the nearest cent)?	$108.24

6 points each		
11	Solve for x: $\log 9+\log 16=\log_x 12$	$\sqrt{10}$
12	Find the sum of all real roots of the equation : $9^{7x^3+3x^2+27x+3}=81^{5x^3-3x^2+17x+11}$	-1
13	Find all real solutions to the equation: $4^x+2^x=8$	$\log_2\left(\frac{\sqrt{33}-1}{2}\right)$ or equiv.
14	If the real solution to the equation $36^x-6\left(24^x\right)-16^{x+1}=0$ can be written in the form $x=\log_y 8$, find the value of y.	$\frac{3}{2}$ or 1.5
15	Find the sum of the common logarithms of all positive integral factors of 100,000.	90

2014 – 2015 Log1 Contest Round 2
Mu Logarithms & Exponents

Name: ________________

	4 points each	
1	Simplify: $\frac{(\log 243)(\log 625)(\log 216)}{(\log 36)(\log 729)(\log 25)}$	$\frac{5}{2}$ or 2.5
2	Solve for the greater value of *x*: $2^{8x-x^2}=\sqrt{128\sqrt{256\sqrt{16}}}$	$4+\sqrt{10}$
3	Find the slope of the tangent to the graph of $y=(\log_2 e)(2^x)$ at the point whose *y*-coordinate is $\log_2\sqrt{e}$.	$\frac{1}{2}$ or 0.5
4	Evaluate: $\left\|e^{5\pi i}\right\|$	1
5	Given that $f(x)=4\cdot 2^{x+2}$, find the value of $f^{-1}(128)$.	3

	5 points each	
6	Find the slope of the tangent to the graph of $y=e^{\sin x}+\log_{\pi/2} x$ at the point $\left(\frac{\pi}{2},e+1\right)$.	$\frac{2}{\pi\ln\left(\frac{\pi}{2}\right)}$ or equiv.
7	Given that $\log 3=a$, $\log 5=b$, and $\log 7=c$, find $(\log_{21}15)(\log_3 7)(\log_{49}5)(\log_5 9)$ in terms of a, b, and c.	$\frac{a+b}{a+c}$
8	If $2^{3x+1}=4^{2a}$ and $7^{6x+3}=\frac{1}{7^{5a}}$, find the numerical value of $\frac{x}{a}$.	$\frac{17}{3}$
9	The number of bacteria in a culture grows exponentially with growth constant 0.05/hour, where the exponential function has base *e*. To the nearest hour, how long does it take for the culture to double its initial population?	14
10	A bank offers an interest-bearing savings account at 4% annual interest, compounded either annually or semiannually. I opened two accounts, one with each type of compounding. I made an initial deposit of \$100 into each account, and I made no deposits to or withdrawals from either account for two years. If the amount of money in each account is rounded down to the nearest cent, then how much more money would I have in the semiannually compounded account after those two years?	\$0.08 or 8 cents

6 points each		
11	Solve for x: $\log 9+\log 16=\log_x 12$	$\sqrt{10}$
12	Find the sum of all imaginary roots of the equation : $9^{7x^3+3x^2+27x+3}=81^{5x^3-3x^2+17x+11}$	4
13	Find the sum of the common logarithms of all positive integral factors of 100,000.	90
14	Evaluate: $\int_0^{\frac{\pi}{2}}\cos^4\theta d\theta$	$\frac{3\pi}{16}$
15	If the area enclosed by the graphs of $y=2^x$, $y=\log_2 x$, $x=1$, and $x=2$ can be written in the form $\log_2 y$, find the value of y.	$\frac{e^3}{4}$

2014 – 2015 Log1 Contest Round 2
Logarithms & Exponents Solutions

Mu	Al	Th	Solution
1	1	1	$\frac{(\log 243)(\log 625)(\log 216)}{(\log 36)(\log 729)(\log 25)} = \frac{(5\log 3)(2\log 25)(3\log 6)}{(2\log 6)(6\log 3)(\log 25)} = \frac{5\cdot 2\cdot 3}{2\cdot 6} = \frac{5}{2}$
	2	2	$\log_2\left(2^{3^4}\cdot 4^{3^2}\right) = \log_2\left(2^{81}\cdot 4^9\right) = \log_2\left(2^{81}\cdot 2^{18}\right) = \log_2\left(2^{99}\right) = 99$
		3	Each term in the series equals 1, so we only need to determine how many terms are in the series. Since the first 1 has no exponents and the last 1 has 7 exponents, there must be a total of eight 1s in the series, so the sum is 8.
2	3		$2^{8x-x^2} = \sqrt{128\sqrt{256\sqrt{16}}} = \sqrt{128\sqrt{1024}} = \sqrt{4096} = 64 = 2^6 \Rightarrow 8x - x^2 = 6 \Rightarrow 0 = x^2 - 8x + 6$ $\Rightarrow x = \frac{8\pm\sqrt{8^2 - 4\cdot 1\cdot 6}}{2} = \frac{8\pm 2\sqrt{10}}{2} = 4\pm\sqrt{10}$, so the greater solution is $4+\sqrt{10}$.
3			Since $y = (\log_2 e)(2^x)$, $y' = (\log_2 e)(2^x \ln 2) = 2^x$. Further, if the y-coordinate is $\log_2\sqrt{e}$ $= (\log_2 e)\left(\frac{1}{2}\right)$, then $2^x = \frac{1}{2}$, making the slope of the tangent $\frac{1}{2}$.
	4	4	$\frac{\log_7\left(343^{\sqrt{2}}\right)}{\log_{81}\left(27^4\right)} = \frac{\sqrt{2}\log_7 343}{\log_{3^4} 27^4} = \frac{\sqrt{2}\cdot 3}{\log_3 27} = \frac{\sqrt{2}\cdot 3}{3} = \sqrt{2}$, so because this is the diagonal length of the square, the sides of the square must be 1, making the enclosed area 1.
		5	Since $\log_3 1 = 0$, the first three numbers of the product, which are all positive real numbers, have a product being multiplied by 0, so the product is 0.
4	5		$\left\|e^{5\pi i}\right\| = \left\|\cos(5\pi) + i\sin(5\pi)\right\| = \left\|-1+0i\right\| = \left\|-1\right\| = 1$
5	6	6	$128 = 4\cdot 2^{f^{-1}(128)+2} \Rightarrow 32 = 2^{f^{-1}(128)+2} \Rightarrow 5 = f^{-1}(128) + 2 \Rightarrow f^{-1}(128) = 3$
6			Since $y = e^{\sin x} + \log_{\pi/2} x$, $y' = e^{\sin x}\cos x + \frac{1}{x\ln\left(\frac{\pi}{2}\right)}$, making the slope of the tangent at the given point $y'\big\|_{x=\pi/2} = e^1\cdot 0 + \frac{1}{\frac{\pi}{2}\ln\left(\frac{\pi}{2}\right)} = \frac{2}{\pi\ln\left(\frac{\pi}{2}\right)}$.
		7	$\log_3\left(9^{e^{\ln\left(e^{\ln 5}\right)}}\right) = \log_3\left(9^{e^{\ln 5}}\right) = \log_3\left(9^5\right) = \log_3\left(3^{10}\right) = 10$
7	7		$(\log_{21} 15)(\log_3 7)(\log_{49} 5)(\log_5 9) = \left(\frac{\log 15}{\log 21}\right)\left(\frac{\log 7}{\log 3}\right)\left(\frac{\log 5}{\log 49}\right)\left(\frac{\log 9}{\log 5}\right) = \left(\frac{\log 3 + \log 5}{\log 3 + \log 7}\right)$ $\left(\frac{\log 7}{\log 3}\right)\left(\frac{\log 5}{2\log 7}\right)\left(\frac{2\log 3}{\log 5}\right) = \frac{\log 3 + \log 5}{\log 3 + \log 7} = \frac{a+b}{a+c}$.

8	8	8	Since $2^{3x+1}=4^{2a}=2^{4a}$, $3x+1=4a$. Since $7^{6x+3}=\frac{1}{7^{5a}}=7^{-5a}$, $6x+3=-5a$. From the first equation, $3x=4a-1\Rightarrow 6x=8a-2$. Plugging this into the second equation, $8a-2+3=-5a\Rightarrow a=-\frac{1}{13}$. Plugging this value into the first equation, $3x+1=4\left(-\frac{1}{13}\right)$ $\Rightarrow x=-\frac{17}{39}$. Therefore, $\frac{x}{a}=\frac{-17/39}{-1/13}=\frac{17}{39}\cdot 13=\frac{17}{3}$.
		9	Plugging in $N=2N_0$ to the equation yields $\ln 2=0.05t\Rightarrow t=20\ln 2$. Using the approximation $\ln 2\approx 0.693$, the amount of time required to double the initial population is $t=20\ln 2\approx 20\cdot 0.693=13.86$ hours, which rounded to the nearest hour means it takes 14 hours to double.
9	9		The equation for this growth model would be $\ln\left(\frac{N}{N_0}\right)=0.05t$, where N_0 is the initial population of the bacteria culture and N is the population of the culture after t hours (units suggested by the units given in the problem). Plugging in $N=2N_0$ to the equation yields $\ln 2=0.05t\Rightarrow t=20\ln 2$. Using the approximation $\ln 2\approx 0.693$, the amount of time required to double the initial population is $t=20\ln 2\approx 20\cdot 0.693$ $=13.86$ hours, which rounded to the nearest hour means it takes 14 hours to double.
	10	10	The amount in the account after two years would be $A=100\left(1+\frac{.04}{2}\right)^{2\cdot 2}=100(1.02)^4$ $=108.243216$, so rounded down to the nearest cent, the account has \$108.24.
10			The amount in the semiannually compounded account after two years would be $A=100\left(1+\frac{.04}{2}\right)^{2\cdot 2}=100(1.02)^4=108.243216$, so rounded down to the nearest cent, that account has \$108.24. The amount in the annually compounded account after two years would be $A=100\left(1+\frac{.04}{1}\right)^{1\cdot 2}=100(1.04)^2=108.16$, so that account has \$108.16. Therefore, the semiannually compounded account has $\$108.24-\$108.16=\$0.08$ more in it.
11	11	11	$\log_{\sqrt{10}}12=\log 12^2=\log 144=\log 9+\log 16=\log_x 12\Rightarrow x=\sqrt{10}$
		12	$9^{7x^3+3x^2+27x+3}=81^{5x^3-3x^2+17x+11}=9^{10x^3-6x^2+34x+22}\Rightarrow 7x^3+3x^2+27x+3=10x^3-6x^2+34x+22$ $\Rightarrow 0=3x^3-9x^2+7x+19=(x+1)(3x^2-12x+19)$, so all three roots of the polynomial are distinct, and the sum of the roots is $-\frac{-9}{3}=3$.
	12		$9^{7x^3+3x^2+27x+3}=81^{5x^3-3x^2+17x+11}=9^{10x^3-6x^2+34x+22}\Rightarrow 7x^3+3x^2+27x+3=10x^3-6x^2+34x+22$ $\Rightarrow 0=3x^3-9x^2+7x+19=(x+1)(3x^2-12x+19)$, and the roots of the quadratic are imaginary, so the sum of the real roots is -1.
12			$9^{7x^3+3x^2+27x+3}=81^{5x^3-3x^2+17x+11}=9^{10x^3-6x^2+34x+22}\Rightarrow 7x^3+3x^2+27x+3=10x^3-6x^2+34x+22$ $\Rightarrow 0=3x^3-9x^2+7x+19=(x+1)(3x^2-12x+19)$, and the roots of the quadratic are imaginary, so the sum of the imaginary roots is $-\frac{-12}{3}=4$.

		13	Since $y=4^x+2^x$ is a continuous increasing function, $y(1)=6$, and $y(2)=20$, the solution must be greater than 1 (since we are looking to solve $y(x)=8$).
	13		$8=4^x+2^x=(2^x)^2+2^x \Rightarrow 0=(2^x)^2+2^x-8 \Rightarrow 2^x=\frac{-1\pm\sqrt{1^2-4\cdot1\cdot-8}}{2\cdot1}=\frac{-1\pm\sqrt{33}}{2}$. However, there is no real solution to $2^x=\frac{-1-\sqrt{33}}{2}$, so the only real solution is $2^x=\frac{-1+\sqrt{33}}{2} \Rightarrow x=\log_2\left(\frac{\sqrt{33}-1}{2}\right)$.
	14	14	$0=36^x-6(24^x)-16^{x+1}=(6^x)^2-6(6^x4^x)-16(4^x)^2=(6^x-8\cdot4^x)(6^x+2\cdot4^x)$, and there is no real solution to the second factor equals 0. Therefore, $0=6^x-8\cdot4^x \Rightarrow 6^x=8\cdot4^x$ $\Rightarrow \left(\frac{3}{2}\right)^x=8 \Rightarrow x=\log_{3/2}8$, so $y=\frac{3}{2}$.
13	15	15	Since $100{,}000=10^5=2^5\cdot5^5$, it has $(5+1)(5+1)=36$ positive integral factors. Further, each positive integral factor can be paired with the factor whose product with it is 100,000 when combining the logarithms—for example, $\log1+\log100{,}000$ $=\log100{,}000$; $\log2+\log50{,}000=\log100{,}000$; and so on. Since there are 36 positive integral factors, and since 100,000 is not a perfect square, there are 18 such pairs of distinct positive integral factors, each of whose corresponding sum is $\log100{,}000=5$. Therefore, the sum is $18\cdot5=90$.
14			Using Wallis' integral formula, $\int_0^{\frac{\pi}{2}}\cos^4\theta d\theta=\frac{\pi}{2}\cdot\frac{1}{2}\cdot\frac{3}{4}=\frac{3\pi}{16}$ <u>OR</u> $\int\cos^4\theta d\theta=\frac{1}{4}\int(1+\cos2\theta)^2d\theta=\frac{1}{4}\int(1+2\cos2\theta+\cos^22\theta)d\theta$ $=\frac{1}{4}\int\left(1+2\cos2\theta+\frac{1}{2}(1+\cos4\theta)\right)d\theta=\frac{1}{4}\int\left(\frac{3}{2}+2\cos2\theta+\frac{1}{2}\cos4\theta\right)d\theta$ $=\frac{1}{4}\left(\frac{3}{2}\theta+\sin2\theta+\frac{1}{8}\sin4\theta\right)+C=\frac{3}{8}\theta+\frac{1}{4}\sin2\theta+\frac{1}{32}\sin4\theta+c$, so $\int_0^{\frac{\pi}{2}}\cos^4\theta d\theta$ $=\left(\frac{3}{8}\cdot\frac{\pi}{2}+\frac{1}{4}\sin\pi+\frac{1}{32}\sin2\pi\right)-\left(\frac{3}{8}\cdot0+\frac{1}{4}\sin0+\frac{1}{32}\sin0\right)=\frac{3\pi}{16}$.
15			Based on the graphs, the area would be $\int_1^2(2^x-\log_2x)dx=\int_1^2\left(2^x-\frac{\ln x}{\ln2}\right)dx$ $=\int_1^2 2^xdx-\frac{1}{\ln2}\int_1^2\ln xdx=\frac{2^x}{\ln2}\Big\vert_1^2-\frac{x\ln x-x}{\ln2}\Big\vert_1^2=\left(\frac{2^2-2^1}{\ln2}\right)-\left(\frac{(2\ln2-2)-(1\ln1-1)}{\ln2}\right)$ $=\frac{2}{\ln2}-\frac{2\ln2-1}{\ln2}=\frac{3-2\ln2}{\ln2}=\frac{\ln e^3-\ln4}{\ln2}=\frac{\ln\left(e^3/4\right)}{\ln2}=\log_2\left(\frac{e^3}{4}\right)$, so $y=\frac{e^3}{4}$.