2014 – 2015 Log1 Contest Round 1
Theta Circles & Polygons

Name: _____

4 points each		
1	Find the area, in square inches, enclosed by a circle whose diameter is 8 inches.	
2	A rectangle has sides of length 4 and 6. Find the area enclosed by the rectangle's circumscribed circle.	
3	An equilateral triangle has a perimeter of 6. Find the enclosed area of this triangle.	
4	A cube with 12-inch edge length is divided into cubes with 4-inch edge lengths. What is the maximum number of these 4-inch edge length cubes?	
5	Find the perimeter, in inches, of a square that encloses an area of 12 square inches.	

5 points each		
6	A rectangle has dimensions such that its perimeter and enclosed area are numerically equivalent. If the length of the rectangle is 8 inches, find the width, in inches, of the rectangle.	
7	Find the perimeter of a regular hexagon whose enclosed area is $54\sqrt{3}$ square units.	
8	Find the total number of diagonals in a regular icosagon.	
9	A square is inscribed in a circle. Find the ratio of the radius of the circle to the apothem of the square.	
10	Two circles with radii of lengths 3 and 6 inches have centers that are $4+\sqrt{51}$ inches apart. Find the distance, in inches, between the points of tangency of a common external tangent of the two circles.	

6 points each		
11	Find the perimeter, in inches, of a regular gigagon with side length 0.00048 inches.	
12	A kite with diagonals of lengths 20 and 22 inches is inscribed in a circle. Find the area, in square inches, enclosed by the circle.	
13	Sixteen dots are positioned in a 4×4 rectangular array where each dot is one inch away from any adjacent dot either above, below, to the left, or to the right of it. If three dots are to be chosen to form a triangle, in how many ways can this be done?	
14	A sheet of butcher paper 36 inches wide is to have eight plates, each of which has diameter 12 inches, traced onto it. The plates' outlines cannot overlap except in a single point. What is the minimum length, in inches, of paper needed to accommodate the eight plates' outlines?	
15	How many sides does an icosikaienneagon have?	

2014 – 2015 Log1 Contest Round 1
Alpha Circles & Polygons

Name: _____

4 points each		
1	Find the area, in square inches, enclosed by a circle whose diameter is 8 inches.	
2	A rectangle has sides of length 4 and 6. Find the area enclosed by the rectangle's circumscribed circle.	
3	An equilateral triangle has a perimeter of 6. Find the enclosed area of this triangle.	
4	A cube with 12-inch edge length is divided into cubes with 4-inch edge lengths. What is the maximum number of these 4-inch edge length cubes?	
5	Find the surface area, in square inches, of a cube whose volume is 128 cubic inches.	

5 points each		
6	A rectangle has dimensions with integer values such that its perimeter and enclosed area are numerically equivalent. Find the sum of the perimeters of all such rectangles.	
7	Find the perimeter of a regular hexagon whose enclosed area is $54\sqrt{3}$ square units.	
8	Find the total number of diagonals in a regular icosagon.	
9	A circle has two perpendicular chords. The first chord divides the second chord into two segments whose lengths are x and $x^2-12x+48$. The second chord divides the first chord into two segments whose lengths are 4 and 16. Find the area enclosed by the circle.	
10	Find the perimeter, in inches, of a regular gigagon with side length 0.00048 inches.	

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13	A region's boundary is formed by connecting the points $(-4,0)$, $(-2,3)$, $(2,3)$, and $(4,0)$, in that order, with line segments. This region is then revolved about the *x*-axis. Find the volume of this solid formed by this revolution.	
14	A sheet of butcher paper 36 inches wide is to have eight plates, each of which has diameter 12 inches, traced onto it. The plates' outlines cannot overlap except in a single point. What is the minimum length, in inches, of paper needed to accommodate the eight plates' outlines?	
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2014 – 2015 Log1 Contest Round 1
Mu Circles & Polygons

Name: _____

4 points each		
1	Find the area, in square inches, enclosed by a circle whose diameter is 8 inches.	
2	A rectangle has sides of length 4 and 6. Find the area enclosed by the rectangle's circumscribed circle.	
3	An equilateral triangle has a perimeter of 6. Find the enclosed area of this triangle.	
4	Find the slope of the tangent line to the graph of $x^2 + y^2 - 16x + 12y - 300 = 0$ at the point $(-4,10)$.	
5	A cube with 12-inch edge length is divided into cubes with 4-inch edge lengths, and then these 4-inch edge length cubes are divided into cubes with 1-inch edge lengths. What is the maximum number of these 1-inch edge length cubes?	

5 points each		
6	A rectangle has dimensions with integer values such that its perimeter and enclosed area are numerically equivalent. Find the sum of the perimeters of all such rectangles.	
7	Find the perimeter of a regular hexagon whose enclosed area is $54\sqrt{3}$ square units.	
8	Find the value when the total number of diagonals in a regular dodecagon is divided by the total number of diagonals in a regular nonagon.	
9	A circle has two perpendicular chords. The first chord divides the second chord into two segments whose lengths are x and $x^2 - 12x + 48$. The second chord divides the first chord into two segments whose lengths are 4 and 16. Find the area enclosed by the circle.	
10	Find the perimeter, in inches, of a regular gigagon with side length 0.00048 inches.	

6 points each		
11	A kite with diagonals of lengths 20 and 22 inches is inscribed in a circle. Find the area, in square inches, enclosed by the circle.	
12	A region's boundary is formed by connecting the points $(-4,0)$, $(-2,3)$, $(2,3)$, and $(4,0)$, in that order, with line segments. This region is then revolved about the *x*-axis. Find the volume of this solid formed by this revolution.	
13	A farmer wants to use 400 feet of fencing to fence in a triangular region against a straight river. No fencing is necessary along the river, so essentially the farmer is building two walls—one end of each wall is at the edge of the river, the other ends of the walls meet at each other. What is the maximum area, in square feet, the farmer could enclose in this way?	
14	A sheet of butcher paper 36 inches wide is to have eight plates, each of which has diameter 12 inches, traced onto it. The plates' outlines cannot overlap except in a single point. What is the minimum length, in inches, of paper needed to accommodate the eight plates' outlines?	
15	How many sides does an icosikaienneagon have?	

2014 – 2015 Log1 Contest Round 1
Theta Circles & Polygons

Name: _____

4 points each		
1	Find the area, in square inches, enclosed by a circle whose diameter is 8 inches.	16π
2	A rectangle has sides of length 4 and 6. Find the area enclosed by the rectangle's circumscribed circle.	13π
3	An equilateral triangle has a perimeter of 6. Find the enclosed area of this triangle.	$\sqrt{3}$
4	A cube with 12-inch edge length is divided into cubes with 4-inch edge lengths. What is the maximum number of these 4-inch edge length cubes?	27
5	Find the perimeter, in inches, of a square that encloses an area of 12 square inches.	$8\sqrt{3}$

5 points each		
6	A rectangle has dimensions such that its perimeter and enclosed area are numerically equivalent. If the length of the rectangle is 8 inches, find the width, in inches, of the rectangle.	$\frac{8}{3}$
7	Find the perimeter of a regular hexagon whose enclosed area is $54\sqrt{3}$ square units.	36
8	Find the total number of diagonals in a regular icosagon.	170
9	A square is inscribed in a circle. Find the ratio of the radius of the circle to the apothem of the square.	$\sqrt{2}$
10	Two circles with radii of lengths 3 and 6 inches have centers that are $4+\sqrt{51}$ inches apart. Find the distance, in inches, between the points of tangency of a common external tangent of the two circles.	$\sqrt{34}+2\sqrt{6}$

6 points each		
11	Find the perimeter, in inches, of a regular gigagon with side length 0.00048 inches.	480,000
12	A kite with diagonals of lengths 20 and 22 inches is inscribed in a circle. Find the area, in square inches, enclosed by the circle.	121π
13	Sixteen dots are positioned in a 4×4 rectangular array where each dot is one inch away from any adjacent dot either above, below, to the left, or to the right of it. If three dots are to be chosen to form a triangle, in how many ways can this be done?	516
14	A sheet of butcher paper 36 inches wide is to have eight plates, each of which has diameter 12 inches, traced onto it. The plates' outlines cannot overlap except in a single point. What is the minimum length, in inches, of paper needed to accommodate the eight plates' outlines?	$12+12\sqrt{3}$
15	How many sides does an icosikaienneagon have?	29

2014 – 2015 Log1 Contest Round 1
Alpha Circles & Polygons

Name: _____

4 points each		
1	Find the area, in square inches, enclosed by a circle whose diameter is 8 inches.	16π
2	A rectangle has sides of length 4 and 6. Find the area enclosed by the rectangle's circumscribed circle.	13π
3	An equilateral triangle has a perimeter of 6. Find the enclosed area of this triangle.	$\sqrt{3}$
4	A cube with 12-inch edge length is divided into cubes with 4-inch edge lengths. What is the maximum number of these 4-inch edge length cubes?	27
5	Find the surface area, in square inches, of a cube whose volume is 128 cubic inches.	$96\sqrt[3]{4}$

5 points each		
6	A rectangle has dimensions with integer values such that its perimeter and enclosed area are numerically equivalent. Find the sum of the perimeters of all such rectangles.	34
7	Find the perimeter of a regular hexagon whose enclosed area is $54\sqrt{3}$ square units.	36
8	Find the total number of diagonals in a regular icosagon.	170
9	A circle has two perpendicular chords. The first chord divides the second chord into two segments whose lengths are x and $x^2-12x+48$. The second chord divides the first chord into two segments whose lengths are 4 and 16. Find the area enclosed by the circle.	136π
10	Find the perimeter, in inches, of a regular gigagon with side length 0.00048 inches.	480,000

6 points each		
11	A kite with diagonals of lengths 20 and 22 inches is inscribed in a circle. Find the area, in square inches, enclosed by the circle.	121π
12	Sixteen dots are positioned in a 4×4 rectangular array where each dot is one inch away from any adjacent dot either above, below, to the left, or to the right of it. If three dots are to be chosen to form a triangle, in how many ways can this be done?	516
13	A region's boundary is formed by connecting the points $(-4,0)$, $(-2,3)$, $(2,3)$, and $(4,0)$, in that order, with line segments. This region is then revolved about the *x*-axis. Find the volume of this solid formed by this revolution.	48π
14	A sheet of butcher paper 36 inches wide is to have eight plates, each of which has diameter 12 inches, traced onto it. The plates' outlines cannot overlap except in a single point. What is the minimum length, in inches, of paper needed to accommodate the eight plates' outlines?	$12+12\sqrt{3}$
15	How many sides does an icosikaienneagon have?	29

2014 – 2015 Log1 Contest Round 1
Mu Circles & Polygons

Name: _____

4 points each		
1	Find the area, in square inches, enclosed by a circle whose diameter is 8 inches.	16π
2	A rectangle has sides of length 4 and 6. Find the area enclosed by the rectangle's circumscribed circle.	13π
3	An equilateral triangle has a perimeter of 6. Find the enclosed area of this triangle.	$\sqrt{3}$
4	Find the slope of the tangent line to the graph of $x^2+y^2-16x+12y-300=0$ at the point $(-4,10)$.	$\frac{3}{4}$
5	A cube with 12-inch edge length is divided into cubes with 4-inch edge lengths, and then these 4-inch edge length cubes are divided into cubes with 1-inch edge lengths. What is the maximum number of these 1-inch edge length cubes?	1728

5 points each		
6	A rectangle has dimensions with integer values such that its perimeter and enclosed area are numerically equivalent. Find the sum of the perimeters of all such rectangles.	34
7	Find the perimeter of a regular hexagon whose enclosed area is $54\sqrt{3}$ square units.	36
8	Find the value when the total number of diagonals in a regular dodecagon is divided by the total number of diagonals in a regular nonagon.	2
9	A circle has two perpendicular chords. The first chord divides the second chord into two segments whose lengths are x and $x^2-12x+48$. The second chord divides the first chord into two segments whose lengths are 4 and 16. Find the area enclosed by the circle.	136π
10	Find the perimeter, in inches, of a regular gigagon with side length 0.00048 inches.	480,000

6 points each		
11	A kite with diagonals of lengths 20 and 22 inches is inscribed in a circle. Find the area, in square inches, enclosed by the circle.	121π
12	A region's boundary is formed by connecting the points $(-4,0)$, $(-2,3)$, $(2,3)$, and $(4,0)$, in that order, with line segments. This region is then revolved about the *x*-axis. Find the volume of this solid formed by this revolution.	48π
13	A farmer wants to use 400 feet of fencing to fence in a triangular region against a straight river. No fencing is necessary along the river, so essentially the farmer is building two walls—one end of each wall is at the edge of the river, the other ends of the walls meet at each other. What is the maximum area, in square feet, the farmer could enclose in this way?	20,000
14	A sheet of butcher paper 36 inches wide is to have eight plates, each of which has diameter 12 inches, traced onto it. The plates' outlines cannot overlap except in a single point. What is the minimum length, in inches, of paper needed to accommodate the eight plates' outlines?	$12+12\sqrt{3}$
15	How many sides does an icosikaienneagon have?	29

2014 – 2015 Log1 Contest Round 1
Circles & Polygons Solutions

Mu	Al	Th	Solution
1	1	1	The radius is 4 inches, so the area enclosed is $\pi(4)^2 = 16\pi$ square inches.
2	2	2	Because of symmetry, the center of the circle must lie halfway between opposite sides of the rectangle. Therefore, $r^2 = 2^2 + 3^2 = 13$, making the enclosed area 13π.
3	3	3	Each side of the triangle would have length 2, so the enclosed area is $\frac{2^2\sqrt{3}}{4} = \sqrt{3}$.
	4	4	Since the division would create three cubes corresponding to each edge (dimension), there should be a total of $3^3 = 27$ smaller cubes.
4			Differentiating implicitly, $2x + 2y\frac{dy}{dx} - 16 + 12\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{8-x}{y+6} \Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=(-4,10)}$ $= \frac{8-(-4)}{10+6} = \frac{12}{16} = \frac{3}{4}$.
		5	Let *s* be the side length of the square. $s^2 = 12 \Rightarrow s = 2\sqrt{3} \Rightarrow 4s = 8\sqrt{3}$
	5		Let *s* be the edge length of the cube. $s^3 = 128 \Rightarrow s = 4\sqrt[3]{2} \Rightarrow 6s^2 = 96\sqrt[3]{4}$
5			This is essentially the same as dividing the original cube into 1-inch side length cubes, so since the division would create twelve cubes corresponding to each edge (dimension), there should be a total of $12^3 = 1728$ smaller cubes.
		6	Let *l* and *w* be the length and width, respectively, of the rectangle. Since $2l + 2w = lw$, $16 + 2w = 8w \Rightarrow 16 = 6w \Rightarrow w = \frac{8}{3}$.
6	6		Let *l* and *w* be the length and width, respectively, of the rectangle. Since $2l + 2w = lw$, $w = \frac{2l}{l-2}$. The only integer values of *l* that also make *w* an integer are 3, 4, and 6, and the corresponding values of *w* are 6, 4, and 3, respectively. Therefore, the only rectangles that fit the description are a 3×6 rectangle and a 4×4 rectangle (squares are also rectangles). The two perimeters of these rectangles are 18 and 16, so the sum of those perimeters is 34.
7	7	7	Let *s* be the side length of the hexagon. $\frac{3\sqrt{3}}{2}s^2 = 54\sqrt{3} \Rightarrow s^2 = 36 \Rightarrow s = 6 \Rightarrow 6s = 36$
	8	8	An icosagon has 20 sides, so the total number of diagonals is $\frac{20(20-3)}{2} = 170$.
8			A dodecagon has 12 sides, so its total number of diagonals is $\frac{12(12-3)}{2} = 54$. A nonagon has 9 sides, so its total number of diagonals is $\frac{9(9-3)}{2} = 27$. Dividing the first number by the second number yields 2.

		9	Let *s* be the length of a side of the square. The diameter of the circle is the diagonal of the square, so if *r* is the length of the radius, $r=\frac{s\sqrt{2}}{2}$. The apothem of the square is half the length of a side, so if *a* is the length of the apothem, $a=\frac{s}{2}$. Therefore, $\frac{r}{a}=\sqrt{2}$.
9	9		Because the two chords intersect, $x\left(x^2-12x+48\right)=4(16)\Rightarrow 0=x^3-12x^2+48x-64=(x-4)^3\Rightarrow x=4$, so the two chords are both divided into pieces of lengths 4 and 16. Reflect one chord across the diameter parallel to it, so the distance between the chord and its reflection is 12, making the distance from the center of the circle to one of those chords 6. Further, half of one of those chords has length 10, so the radius of the circle is $r=\sqrt{6^2+10^2}=\sqrt{136}$, making the enclosed area 136π.
		10	The lengths of the two circles' radii differ by 3, so if *d* is the distance between the two points of tangency, $d=\sqrt{\left(4+\sqrt{51}\right)^2-3^2}=\sqrt{58+8\sqrt{51}}=\sqrt{34}+\sqrt{24}=\sqrt{34}+2\sqrt{6}$.
10	10	11	A gigagon has 1,000,000,000 sides, so because each side has length 0.00048 inches, the perimeter is $1{,}000{,}000{,}000\cdot 0.00048=480{,}000$.
11	11	12	A kite inscribed in a circle must be symmetric about its longer diagonal, making the longer diagonal a diameter of the circle. Therefore, the radius has length 11 and the enclosed area is $\pi(11)^2=121\pi$.
	12	13	The total number of ways of choosing three of the 16 points is $\binom{16}{3}=560$. However, there are 10 rows of 4 dots, so choosing any 3 collinear dots won't make a triangle; there are a total of $10\binom{4}{3}=40$ ways to choose those. Additionally, there are 4 rows of 3 dots, so choosing any of those rows' dots won't make a triangle. Therefore, the total number of ways to make a triangle is $560-40-4=516$.
12	13		The result of this revolution is a cylinder with radius 3 and height 4 in the middle, capped on both ends with a cone with radius 3 and height 2. Each cone has volume $\frac{1}{3}\pi\left(3^2\right)(2)=6\pi$ and the cylinder has volume $\pi(3)^2(4)=36\pi$, so the total volume is $2(6\pi)+36\pi=48\pi$.
13			If one wall has length *x*, the other wall has length $400-x$. Let θ be the angle between the two walls. Therefore, the area of the region is $\frac{1}{2}x(400-x)\sin\theta$. Since $0^\circ<\theta<180^\circ$, all other things being equal, a value of $\theta=90^\circ$ would give a greatest sine value (1). To maximize $\frac{1}{2}x(400-x)$, recognize that $y=\frac{1}{2}x(400-x)$ is a parabola opening downward with *x*-intercepts of 0 and 400, so its vertex must occur when $x=200$ and is a maximum. Therefore, the maximum area occurs when $x=200$ and $\theta=90^\circ$, making the enclosed area $\frac{1}{2}(200)(400-200)(1)=20{,}000$.

14	14	14	Nine plates could be traced in three rows, tangent to all edges, so a sheet not longer than 36 inches would suffice. However, the most efficient packing of circular regions in a plane is the one where any three adjacent circles are all externally tangent, creating an equilateral triangle with the three centers of those adjacent circles. Therefore, the shortest length would be three rows of 3, 2, and 3 circles. The length would need to accommodate 2 radii and 2 altitudes of the aforementioned equilateral triangles. Since the sides of the equilateral triangles each have length 12 inches, the minimum length is $2(6)+2\left(6\sqrt{3}\right)=12+12\sqrt{3}$.
15	15	15	Icosi- is the tens prefix for 20, and -ennea- is the ones part meaning 9. -kai- and -gon are just added to the word, so the number of sides is 29.