2013 – 2014 Log1 Contest Round 3
Theta Individual

Name: ________________

	4 points each	
1	0 degrees Celsius is equivalent to 32 degrees Fahrenheit, and 100 degrees Celsius is equivalent to 212 degrees Fahrenheit. The number of degrees Fahrenheit is a first-degree polynomial in terms of the number of degrees Celsius. How many degrees are in the Fahrenheit temperature that is exactly twice its equivalent Celsius temperature?	
2	Write in simplest radical form: $\left(\left(\sqrt[6]{\sqrt[9]{3}}\right)^3\right)^{12}$	
3	If *a* and *d* are real numbers, find the value of $\sum_{k=0}^{10}(a+kd)$.	
4	Find an expression for the inverse function $f^{-1}(x)$ if $f(x)=\frac{e^x}{2}$.	
5	One-half of one-third of one-fourth of one-fifth of one sixth of a number equals 3. Find the number.	

	5 points each	
6	Solve for 2×2 matrix *X*: $X\begin{bmatrix}3 & 2\\5 & 4\end{bmatrix}+\begin{bmatrix}1 & 5\\1 & 2\end{bmatrix}=\begin{bmatrix}7 & 3\\4 & 7\end{bmatrix}$	
7	Find the imaginary solution to the equation $z^2-(1+i)z-2-i=0$, where $i=\sqrt{-1}$.	
8	Mikey was buying some Life$_{TM}$ cereal since he likes it. In the first store he spent half of his money plus \$6 on Life$_{TM}$ cereal. In the second store he spent one-third of his remaining money plus \$6 on Life$_{TM}$ cereal. In the third store he spent one-fourth of his remaining money on Life$_{TM}$ cereal. If Mikey came home with \$6, how much did he have before he started buying Life$_{TM}$ cereal, in dollars?	
9	The three terms in the sequence $\log_{23}512$, $\log_{23}x$, $\log_{23}4802$ form an arithmetic progression. What is the value of *x*?	
10	$\triangle ABC$ has a right angle at *B*. The altitude of the triangle from *B* to $\overline{AC}$ intersects $\overline{AC}$ at point *D*. If $\lvert\overline{AD}\rvert=5$ and $\lvert\overline{CD}\rvert=7$, find the area enclosed by $\triangle ABC$.	

6 points each		
11	Thom Yorke has 10 identical Valentine's Day chocolates that that he is going to distribute to 5 kids, one of whom is Kid A. Kid A cannot receive more than 2 Valentine's Day chocolates, or else he will get a toothache. In how many ways can Thom Yorke distribute the Valentine's Day chocolates? Assume the chocolates are indistinguishable from each other.	
12	A positive integer k is added to each number in the sequence 24, 105, 204 to yield a new sequence, and the new sequence consists of the squares of three consecutive terms of an arithmetic sequence. Find the value of k.	
13	A box contains one chip labeled "A", two chips labeled "B", three chips labeled "C", and so on up to twenty-six chips labeled "Z". If chips are removed from this box one at a time without noting the letter labeled on the chips, what is the minimum number of chips that must be removed from the box in order to guarantee that there are at least ten chips removed with the same label?	
14	The region bounded by the graphs of $y=\sqrt{16-x^2}$ and the x-axis is revolved about the y-axis. Find the volume of the resulting solid.	
15	Find the positive difference between the lengths of the radii of the circumscribed and inscribed circles of the triangle whose side lengths are 7, 8, and 13.	

2013 – 2014 Log1 Contest Round 3
Alpha Individual

Name: ________________

	4 points each	
1	0 degrees Celsius is equivalent to 32 degrees Fahrenheit, and 100 degrees Celsius is equivalent to 212 degrees Fahrenheit. The number of degrees Fahrenheit is a first-degree polynomial in terms of the number of degrees Celsius. How many degrees are in the Fahrenheit temperature that is exactly twice its equivalent Celsius temperature?	
2	If a and d are real numbers, find the value of $\sum_{k=0}^{10}(a+kd)$.	
3	Find an expression for the inverse function $f^{-1}(x)$ if $f(x)=\frac{e^x}{2}$.	
4	One-half of one-third of one-fourth of one-fifth of one sixth of a number equals 3. Find the number.	
5	Solve for 2×2 matrix X: $X\begin{bmatrix}3 & 2\\5 & 4\end{bmatrix}+\begin{bmatrix}1 & 5\\1 & 2\end{bmatrix}=\begin{bmatrix}7 & 3\\4 & 7\end{bmatrix}$	

	5 points each	
6	Find the imaginary solution to the equation $z^2-(1+i)z-2-i=0$, where $i=\sqrt{-1}$.	
7	Mikey was buying some Life$_{TM}$ cereal since he likes it. In the first store he spent half of his money plus \$6 on Life$_{TM}$ cereal. In the second store he spent one-third of his remaining money plus \$6 on Life$_{TM}$ cereal. In the third store he spent one-fourth of his remaining money on Life$_{TM}$ cereal. If Mikey came home with \$6, how much did he have before he started buying Life$_{TM}$ cereal, in dollars?	
8	How many five-digit positive integers are such that all five digits are of the same parity (odd or even)?	
9	The three terms in the sequence $\log_{23}512$, $\log_{23}x$, $\log_{23}4802$ form an arithmetic progression. What is the value of x?	
10	Thom Yorke has 10 identical Valentine's Day chocolates that that he is going to distribute to 5 kids, one of whom is Kid A. Kid A cannot receive more than 2 Valentine's Day chocolates, or else he will get a toothache. In how many ways can Thom Yorke distribute the Valentine's Day chocolates? Assume the chocolates are indistinguishable from each other.	

6 points each		
11	A positive integer k is added to each number in the sequence 24, 105, 204 to yield a new sequence, and the new sequence consists of the squares of three consecutive terms of an arithmetic sequence. Find the value of k.	
12	Given that $0<a<b$ and $a^2+b^2=38ab$, what is the numerical value of $\frac{a+b}{a-b}$?	
13	A box contains one chip labeled "A", two chips labeled "B", three chips labeled "C", and so on up to twenty-six chips labeled "Z". If chips are removed from this box one at a time without noting the letter labeled on the chips, what is the minimum number of chips that must be removed from the box in order to guarantee that there are at least ten chips removed with the same label?	
14	The region bounded by the graphs of $y=\sqrt{16-x^2}$ and the x-axis is revolved about the y-axis. Find the volume of the resulting solid.	
15	A and B are acute angles satisfying $A+B=45^\circ$. If $\tan A=\frac{4}{7}$, find the value of $\tan B$.	

2013 – 2014 Log1 Contest Round 3
Mu Individual

Name: ________________

4 points each		
1	0 degrees Celsius is equivalent to 32 degrees Fahrenheit, and 100 degrees Celsius is equivalent to 212 degrees Fahrenheit. The number of degrees Fahrenheit is a first-degree polynomial in terms of the number of degrees Celsius. How many degrees are in the Fahrenheit temperature that is exactly twice its equivalent Celsius temperature?	
2	If *a* and *d* are real numbers, find the value of $\sum_{k=0}^{10}(a+kd)$.	
3	Find the solution $y=f(x)$ to the differential equation $\frac{dy}{dx}=3x^2-2x+1$, where $y=0$ when $x=1$.	
4	One-half of one-third of one-fourth of one-fifth of one sixth of a number equals 3. Find the number.	
5	Find the tens' digit of the quantity 17^{2014}.	

5 points each		
6	Find the imaginary solution to the equation $z^2-(1+i)z-2-i=0$, where $i=\sqrt{-1}$.	
7	How many five-digit positive integers are such that all five digits are of the same parity (odd or even)?	
8	Find the area enclosed by the curves with equations $y=2x^2-3x$ and $y=-3x^2+2x$.	
9	The three terms in the sequence $\log_{23}512,\ \log_{23}x,\ \log_{23}4802$ form an arithmetic progression. What is the value of *x*?	
10	Thom Yorke has 10 identical Valentine's Day chocolates that that he is going to distribute to 5 kids, one of whom is Kid A. Kid A cannot receive more than 2 Valentine's Day chocolates, or else he will get a toothache. In how many ways can Thom Yorke distribute the Valentine's Day chocolates? Assume the chocolates are indistinguishable from each other.	

6 points each		
11	Given that $0<a<b$ and $a^2+b^2=38ab$, what is the numerical value of $\frac{a+b}{a-b}$?	
12	If $3=\sqrt[3]{9-x}+\sqrt[3]{9+x}$, where $x>0$, find the value of x.	
13	A box contains one chip labeled "A", two chips labeled "B", three chips labeled "C", and so on up to twenty-six chips labeled "Z". If chips are removed from this box one at a time without noting the letter labeled on the chips, what is the minimum number of chips that must be removed from the box in order to guarantee that there are at least ten chips removed with the same label?	
14	Evaluate as a single fraction: $\int_0^1 \frac{x-1}{\sqrt{4-3x^2}}dx$	
15	If $f(x)=ax^3+bx^2$, where a and b are real numbers, has a point of inflection at $(3,6)$, find the numerical value of $a+b$.	

2013 – 2014 Log1 Contest Round 3
Theta Individual

Name: ________________

	4 points each	
1	0 degrees Celsius is equivalent to 32 degrees Fahrenheit, and 100 degrees Celsius is equivalent to 212 degrees Fahrenheit. The number of degrees Fahrenheit is a first-degree polynomial in terms of the number of degrees Celsius. How many degrees are in the Fahrenheit temperature that is exactly twice its equivalent Celsius temperature?	320
2	Write in simplest radical form: $\left(\left(\sqrt[6]{\sqrt[9]{3}}\right)^3\right)^{12}$	$\sqrt[3]{9}$
3	If a and d are real numbers, find the value of $\sum_{k=0}^{10}(a+kd)$.	$11a+55d$
4	Find an expression for the inverse function $f^{-1}(x)$ if $f(x)=\frac{e^x}{2}$.	$\ln(2x)$
5	One-half of one-third of one-fourth of one-fifth of one sixth of a number equals 3. Find the number.	2160

	5 points each	
6	Solve for 2×2 matrix X: $X\begin{bmatrix}3 & 2\\5 & 4\end{bmatrix}+\begin{bmatrix}1 & 5\\1 & 2\end{bmatrix}=\begin{bmatrix}7 & 3\\4 & 7\end{bmatrix}$	$\begin{bmatrix}17 & -9\\-6.5 & 4.5\end{bmatrix}$
7	Find the imaginary solution to the equation $z^2-(1+i)z-2-i=0$, where $i=\sqrt{-1}$.	$2+i$
8	Mikey was buying some Life$_{TM}$ cereal since he likes it. In the first store he spent half of his money plus \$6 on Life$_{TM}$ cereal. In the second store he spent one-third of his remaining money plus \$6 on Life$_{TM}$ cereal. In the third store he spent one-fourth of his remaining money on Life$_{TM}$ cereal. If Mikey came home with \$6, how much did he have before he started buying Life$_{TM}$ cereal, in dollars?	54
9	The three terms in the sequence $\log_{23} 512$, $\log_{23} x$, $\log_{23} 4802$ form an arithmetic progression. What is the value of x?	1568
10	$\triangle ABC$ has a right angle at B. The altitude of the triangle from B to $\overline{AC}$ intersects $\overline{AC}$ at point D. If $\left\lvert\overline{AD}\right\rvert=5$ and $\left\lvert\overline{CD}\right\rvert=7$, find the area enclosed by $\triangle ABC$.	$6\sqrt{35}$

6 points each		
11	Thom Yorke has 10 identical Valentine's Day chocolates that that he is going to distribute to 5 kids, one of whom is Kid A. Kid A cannot receive more than 2 Valentine's Day chocolates, or else he will get a toothache. In how many ways can Thom Yorke distribute the Valentine's Day chocolates? Assume the chocolates are indistinguishable from each other.	671
12	A positive integer k is added to each number in the sequence 24, 105, 204 to yield a new sequence, and the new sequence consists of the squares of three consecutive terms of an arithmetic sequence. Find the value of k.	120
13	A box contains one chip labeled "A", two chips labeled "B", three chips labeled "C", and so on up to twenty-six chips labeled "Z". If chips are removed from this box one at a time without noting the letter labeled on the chips, what is the minimum number of chips that must be removed from the box in order to guarantee that there are at least ten chips removed with the same label?	199
14	The region bounded by the graphs of $y=\sqrt{16-x^2}$ and the x-axis is revolved about the y-axis. Find the volume of the resulting solid.	$\frac{128\pi}{3}$
15	Find the positive difference between the lengths of the radii of the circumscribed and inscribed circles of the triangle whose side lengths are 7, 8, and 13.	$\frac{10\sqrt{3}}{3}$

2013 – 2014 Log1 Contest Round 3
Alpha Individual

Name: ________________

	4 points each	
1	0 degrees Celsius is equivalent to 32 degrees Fahrenheit, and 100 degrees Celsius is equivalent to 212 degrees Fahrenheit. The number of degrees Fahrenheit is a first-degree polynomial in terms of the number of degrees Celsius. How many degrees are in the Fahrenheit temperature that is exactly twice its equivalent Celsius temperature?	320
2	If *a* and *d* are real numbers, find the value of $\sum_{k=0}^{10}(a+kd)$.	$11a+55d$
3	Find an expression for the inverse function $f^{-1}(x)$ if $f(x)=\frac{e^x}{2}$.	$\ln(2x)$
4	One-half of one-third of one-fourth of one-fifth of one sixth of a number equals 3. Find the number.	2160
5	Solve for 2×2 matrix *X*: $X\begin{bmatrix}3 & 2\\5 & 4\end{bmatrix}+\begin{bmatrix}1 & 5\\1 & 2\end{bmatrix}=\begin{bmatrix}7 & 3\\4 & 7\end{bmatrix}$	$\begin{bmatrix}17 & -9\\-6.5 & 4.5\end{bmatrix}$

	5 points each	
6	Find the imaginary solution to the equation $z^2-(1+i)z-2-i=0$, where $i=\sqrt{-1}$.	$2+i$
7	Mikey was buying some Life$_{TM}$ cereal since he likes it. In the first store he spent half of his money plus $6 on Life$_{TM}$ cereal. In the second store he spent one-third of his remaining money plus $6 on Life$_{TM}$ cereal. In the third store he spent one-fourth of his remaining money on Life$_{TM}$ cereal. If Mikey came home with $6, how much did he have before he started buying Life$_{TM}$ cereal, in dollars?	54
8	How many five-digit positive integers are such that all five digits are of the same parity (odd or even)?	5625
9	The three terms in the sequence $\log_{23}512$, $\log_{23}x$, $\log_{23}4802$ form an arithmetic progression. What is the value of *x*?	1568
10	Thom Yorke has 10 identical Valentine's Day chocolates that that he is going to distribute to 5 kids, one of whom is Kid A. Kid A cannot receive more than 2 Valentine's Day chocolates, or else he will get a toothache. In how many ways can Thom Yorke distribute the Valentine's Day chocolates? Assume the chocolates are indistinguishable from each other.	671

6 points each		
11	A positive integer k is added to each number in the sequence 24, 105, 204 to yield a new sequence, and the new sequence consists of the squares of three consecutive terms of an arithmetic sequence. Find the value of k.	120
12	Given that $0<a<b$ and $a^2+b^2=38ab$, what is the numerical value of $\frac{a+b}{a-b}$?	$-\frac{\sqrt{10}}{3}$
13	A box contains one chip labeled "A", two chips labeled "B", three chips labeled "C", and so on up to twenty-six chips labeled "Z". If chips are removed from this box one at a time without noting the letter labeled on the chips, what is the minimum number of chips that must be removed from the box in order to guarantee that there are at least ten chips removed with the same label?	199
14	The region bounded by the graphs of $y=\sqrt{16-x^2}$ and the x-axis is revolved about the y-axis. Find the volume of the resulting solid.	$\frac{128\pi}{3}$
15	A and B are acute angles satisfying $A+B=45^\circ$. If $\tan A=\frac{4}{7}$, find the value of $\tan B$.	$\frac{3}{11}$

2013 – 2014 Log1 Contest Round 3
Mu Individual

Name: ________________

	4 points each	
1	0 degrees Celsius is equivalent to 32 degrees Fahrenheit, and 100 degrees Celsius is equivalent to 212 degrees Fahrenheit. The number of degrees Fahrenheit is a first-degree polynomial in terms of the number of degrees Celsius. How many degrees are in the Fahrenheit temperature that is exactly twice its equivalent Celsius temperature?	320
2	If *a* and *d* are real numbers, find the value of $\sum_{k=0}^{10}\left(a+kd\right)$.	$11a+55d$
3	Find the solution $y=f\left(x\right)$ to the differential equation $\frac{dy}{dx}=3x^2-2x+1$, where $y=0$ when $x=1$.	$y=x^3-x^2$ $+x-1$
4	One-half of one-third of one-fourth of one-fifth of one sixth of a number equals 3. Find the number.	2160
5	Find the tens' digit of the quantity 17^{2014}.	2

	5 points each	
6	Find the imaginary solution to the equation $z^2-\left(1+i\right)z-2-i=0$, where $i=\sqrt{-1}$.	$2+i$
7	How many five-digit positive integers are such that all five digits are of the same parity (odd or even)?	5625
8	Find the area enclosed by the curves with equations $y=2x^2-3x$ and $y=-3x^2+2x$.	$\frac{5}{6}$
9	The three terms in the sequence $\log_{23}512,\ \log_{23}x,\ \log_{23}4802$ form an arithmetic progression. What is the value of *x*?	1568
10	Thom Yorke has 10 identical Valentine's Day chocolates that that he is going to distribute to 5 kids, one of whom is Kid A. Kid A cannot receive more than 2 Valentine's Day chocolates, or else he will get a toothache. In how many ways can Thom Yorke distribute the Valentine's Day chocolates? Assume the chocolates are indistinguishable from each other.	671

6 points each		
11	Given that $0 < a < b$ and $a^2 + b^2 = 38ab$, what is the numerical value of $\frac{a+b}{a-b}$?	$-\frac{\sqrt{10}}{3}$
12	If $3 = \sqrt[3]{9-x} + \sqrt[3]{9+x}$, where $x > 0$, find the value of x.	$4\sqrt{5}$
13	A box contains one chip labeled "A", two chips labeled "B", three chips labeled "C", and so on up to twenty-six chips labeled "Z". If chips are removed from this box one at a time without noting the letter labeled on the chips, what is the minimum number of chips that must be removed from the box in order to guarantee that there are at least ten chips removed with the same label?	199
14	Evaluate as a single fraction: $\int_0^1 \frac{x-1}{\sqrt{4-3x^2}} dx$	$\frac{3-\pi\sqrt{3}}{9}$
15	If $f(x) = ax^3 + bx^2$, where a and b are real numbers, has a point of inflection at $(3,6)$, find the numerical value of $a+b$.	$\frac{8}{9}$

2013 – 2014 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
1	1	1	Using the given information, the relationship is $F=1.8C+32$. Therefore, $F=1.8(0.5F)+32=0.9F+32\Rightarrow 0.1F=32\Rightarrow F=320$.
		2	$\left(\left(\sqrt[6]{\sqrt[9]{3}}\right)^3\right)^{12}=3^{\frac{12\cdot 3}{6\cdot 9}}=3^{\frac{36}{54}}=3^{\frac{2}{3}}=\sqrt[3]{9}$
2	2	3	Since this is an arithmetic series with 11 terms, $\sum_{k=0}^{10}(a+kd)=\frac{11}{2}(a+(a+10d))$ $=\frac{11}{2}(2a+10d)=11a+55d$.
	3	4	$x=\frac{e^{f^{-1}(x)}}{2}\Rightarrow e^{f^{-1}(x)}=2x\Rightarrow f^{-1}(x)=\ln(2x)$
3			$y=\int(3x^2-2x+1)dx=x^3-x^2+x+C$. Plugging in $(1,0)$ yields $0=1^3-1^2+1+C$ $\Rightarrow C=-1$. Therefore, $y=x^3-x^2+x-1$.
4	4	5	$\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{4}\cdot\frac{1}{5}\cdot\frac{1}{6}x=3\Rightarrow x=3\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6=2160$
	5	6	$X\begin{bmatrix}3&2\\5&4\end{bmatrix}+\begin{bmatrix}1&5\\1&2\end{bmatrix}=\begin{bmatrix}7&3\\4&7\end{bmatrix}\Rightarrow X\begin{bmatrix}3&2\\5&4\end{bmatrix}=\begin{bmatrix}6&-2\\3&5\end{bmatrix}\Rightarrow X=\begin{bmatrix}6&-2\\3&5\end{bmatrix}\cdot\frac{1}{2}\begin{bmatrix}4&-2\\-5&3\end{bmatrix}$ $X=\begin{bmatrix}6&-2\\3&5\end{bmatrix}\begin{bmatrix}2&-1\\-2.5&1.5\end{bmatrix}=\begin{bmatrix}17&-9\\-6.5&4.5\end{bmatrix}$
5			Using Euler's theorem, $17^{\phi(100)}=17^{40}\equiv 1(\text{mod}\,100)$. Therefore, $17^{2000}=\left(17^{40}\right)^{50}$ $\equiv 1^{50}=1(\text{mod}\,100)$, so we only need to see what the tens' digit of 17^{14} is. In modulus 100, $17^{14}\equiv 89^7\equiv 21^3\cdot 89\equiv 41\cdot 21\cdot 89\equiv 61\cdot 89\equiv 29$, so the tens' digit is 2.
6	6	7	$z=\frac{1+i\pm\sqrt{(1+i)^2-4(1)(-2-i)}}{2\cdot 1}=\frac{1+i\pm\sqrt{8+6i}}{2}=\frac{1+i\pm(3+i)}{2}=-1$ or $2+i$, so the imaginary solution is $2+i$.
	7	8	Working backwards, if Mikey came home with \$6 after spending one-fourth of his money in the third store, he went into the third store with \$8. In the second store he spent one-third of his money plus \$6, so adding the \$6 yields \$14, and if he spent one-third of his money, he went into the second store with \$21. Finally, he spent half of his money plus \$6 in the first store, so adding the \$6 yields \$27, and if he spent half of his money, he went into the first store with \$54.
7	8		If all five digits are odd, then there are 5 choices for each digit, making a total of $5^5=3125$ numbers with all five digits odd. If all five digits are even, then there are 4 choices for the first digit (0 can't be the first digit) and 5 choices for the other four digits, making a total of $4\cdot 5^4=2500$ numbers with all five digits even. This makes a total of $3125+2500=5625$ such numbers.

8			To find the intersection values, set the equations equal to each other: $2x^2-3x=-3x^2+2x\Rightarrow 0=5x^2-5x=5x(x-1)\Rightarrow x=0$ or $x=1$. Both curves are parabolas, so the one that opens downward will be the "upper" curve, and the area is $\int_0^1\left(\left(-3x^2+2x\right)-\left(2x^2-3x\right)\right)dx=\int_0^1\left(-5x^2+5x\right)dx=\left(-\frac{5}{3}x^3+\frac{5}{2}x^2\right)\Big\vert_0^1=\left(-\frac{5}{3}+\frac{5}{2}\right)-(0+0)$ $=\frac{5}{6}$.
9	9	9	Since $\log_{23}512,\ \log_{23}x,\ \log_{23}4802$ forms an arithmetic progression, $512,\ x,\ 4802$ forms a geometric progression. Therefore, $x=\sqrt{512\cdot 4802}=1568$.
		10	Because the triangle is right, $\left\|\overline{BD}\right\|=\sqrt{5\cdot 7}=\sqrt{35}$. The hypotenuse has length 12, so the enclosed area is $\frac{1}{2}\cdot 12\cdot\sqrt{35}=6\sqrt{35}$.
10	10	11	Examining the distribution casewise, if Kid A gets 0 chocolates, then Thom Yorke distributes 10 chocolates to 4 kids, which can be done in $\binom{4+10-1}{10}=\binom{13}{10}=286$ ways. If Kid A gets 1 chocolate, then Thom Yorke distributes 9 chocolates to 4 kids, which can be done in $\binom{4+9-1}{9}=\binom{12}{9}=220$ ways. If Kid A gets 2 chocolates, then Thom Yorke distributes 8 chocolates to 4 kids, which can be done in $\binom{4+8-1}{8}=\binom{11}{8}$ $=165$ ways. This yields a total of $286+220+165=671$ total distributions.
	11	12	From the given information, $24+k=(a-d)^2=a^2+d^2-2ad$, $105+k=a^2$, and $204+k$ $=(a+d)^2=a^2+d^2+2ad$. Adding the first and third equations yields $228+2k$ $=2a^2+2d^2\Rightarrow 114+k=a^2+d^2$. Plugging in the second equation yields $114+k$ $=105+k+d^2\Rightarrow d^2=9\Rightarrow d=3$ (since $d>0$). Now, taking the third equation and subtracting the first equation yields $180=4ad=12a\Rightarrow a=15$. Therefore, $105+k$ $=15^2=225\Rightarrow k=120$.
11	12		First, since $0<a<b$, $\frac{a+b}{a-b}<0$. Now, if $x=\frac{a+b}{a-b}$, then $x^2=\frac{(a+b)^2}{(a-b)^2}=\frac{a^2+b^2+2ab}{a^2+b^2-2ab}$ $=\frac{38ab+2ab}{38ab-2ab}=\frac{40ab}{36ab}=\frac{10}{9}\Rightarrow x=-\frac{\sqrt{10}}{3}$.
12			Cubing both sides of the equation, $3^3=(9-x)+(9+x)+3\sqrt[3]{81-x^2}\left(\sqrt[3]{9-x}+\sqrt[3]{9+x}\right)$ $\Rightarrow 27=18+9\sqrt[3]{81-x^2}\Rightarrow 1=\sqrt[3]{81-x^2}\Rightarrow x^2=80\Rightarrow x=4\sqrt{5}$.
13	13	13	At worst case, all of the chips labeled with letters from "A" to "I" can be drawn since there are not ten chips labeled with any one of the letters; this is $1+2+\ldots+9=45$ chips. Of the remaining 17 letters, again at worst case, nine of each lettered chip could be drawn, which is $17\cdot 9=153$ chips. Therefore, if one more chip was drawn, there must be 10 chips with one specific letter removed. This makes the minimum number of chips removed $45+153+1=199$.

	14	14	The graph is a semicircle symmetric to the y-axis, so the revolution yields a hemisphere with radius of length 4. Therefore, the volume is $\frac{1}{2}\cdot\frac{4}{3}\pi(4)^3=\frac{128\pi}{3}$.
14			First, split the integral: $\int\frac{x-1}{\sqrt{4-3x^2}}dx=\int\frac{x}{\sqrt{4-3x^2}}dx-\int\frac{1}{\sqrt{4-3x^2}}dx$. For the first integral, use the substitution $u=4-3x^2$, $du=-6xdx$; for the second integral, use the substitution $v=\sqrt{3}x$, $dv=\sqrt{3}dx$. This becomes $-\frac{1}{6}\int\frac{1}{\sqrt{u}}du-\frac{1}{\sqrt{3}}\int\frac{1}{\sqrt{4-v^2}}dv$ $=-\frac{1}{3}\sqrt{u}-\frac{1}{\sqrt{3}}\arcsin\frac{v}{2}=-\frac{1}{3}\sqrt{4-3x^2}-\frac{1}{\sqrt{3}}\arcsin\frac{\sqrt{3}x}{2}$. Therefore, $\int_0^1\frac{x-1}{\sqrt{4-3x^2}}dx$ $=\left(-\frac{1}{3}\sqrt{4-3x^2}-\frac{1}{\sqrt{3}}\arcsin\frac{\sqrt{3}x}{2}\right)\Bigg\vert_0^1=\left(-\frac{1}{3}-\frac{\pi}{3\sqrt{3}}\right)-\left(-\frac{2}{3}-0\right)=\frac{\sqrt{3}-\pi}{3\sqrt{3}}=\frac{3-\pi\sqrt{3}}{9}$.
		15	Using Hero's formula, since the semiperimeter of the triangle is 14, the area enclosed by the triangle is $\sqrt{14(14-7)(14-8)(14-13)}=14\sqrt{3}$. Therefore, the inscribed circle has radius of length $\frac{14\sqrt{3}}{14}=\sqrt{3}$ and the circumscribed circle has radius of length $\frac{7(8)(13)}{4(14\sqrt{3})}=\frac{13\sqrt{3}}{3}$. The positive difference in these lengths is $\frac{13\sqrt{3}}{3}-\sqrt{3}=\frac{10\sqrt{3}}{3}$.
	15		$1=\tan 45^\circ=\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{4}{7}+\tan B}{1-\frac{4}{7}\tan B}\Rightarrow\frac{4}{7}+\tan B=1-\frac{4}{7}\tan B$ $\Rightarrow\frac{11}{7}\tan B=\frac{3}{7}\Rightarrow\tan B=\frac{3}{11}$
15			Since $(3,6)$ is on the graph of f, $6=f(3)=27a+9b\Rightarrow 2=9a+3b$. Additionally, $f'(x)=3ax^2+2bx\Rightarrow f''(x)=6ax+2b$. Since f has a point of inflection at $(3,6)$, $0=f'(3)=18a+2b$. Solving this system of equations yields $a=-\frac{1}{9}$ and $b=1$, so $a+b=\frac{8}{9}$.