2013 – 2014 Log1 Contest Round 2
Theta Complex Numbers

Name: ________________

4 points each		
1	Write in $a+bi$ form: $(-1+i)+(3+5i)-(2-i)$	
2	Write in $a+bi$ form: $(-1+i)(3+5i)(2-i)$	
3	Write in $a+bi$ form: $\frac{6+5i}{-4+16i}$	
4	Evaluate: $\lvert -65+72i \rvert$	
5	Determine if the following statement is always, sometimes, or never true (you may just write "always", "sometimes", or "never" as your answer): "The sum of two real numbers is a complex number."	

5 points each		
6	What is the sum of the magnitudes of the eight complex solutions of the equation $x^8=1$?	
7	Simplify: $\frac{\sqrt{-5}\cdot\sqrt{-15}}{\sqrt{(-5)(-15)}}$	
8	For the polynomial $f(x)=x^3+12x^2+bx+60$, where b is real, one of the roots equals the sum of the other two roots. Find the complex root of f whose imaginary part is positive.	
9	For complex number $z=2-3i$, find the value of $z\cdot\overline{z}$.	
10	Find the distance between the complex numbers $-2+i$ and $3+5i$ in the Argand plane.	

6 points each		
11	Write in $a+bi$ form: $\frac{(1+i)^{17}}{(1-i)^{16}}$	
12	Find the sum of all positive integers $n<100$ such that $\left(\sqrt{3}-i\right)^n$ is a real number.	
13	Find all complex solutions to the equation $ix^2+(1-5i)x-1+8i=0$.	
14	Write in $a+bi$ form: $\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{6}-\sqrt{2}}{4}i\right)^{2014}$	
15	Consider polynomial $f(x)=x^5+ax^4+bx^3+cx^2+dx-442$ with integral coefficients. f has one real root and four imaginary roots. The product of two of the imaginary roots is $11+10i$. Find the value of the real root of f.	

2013 – 2014 Log1 Contest Round 2
Alpha Complex Numbers

Name: ________________

	4 points each	
1	Write in $a+bi$ form: $(-1+i)+(3+5i)-(2-i)$	
2	Write in $a+bi$ form: $(-1+i)(3+5i)(2-i)$	
3	Write in $a+bi$ form: $\frac{6+5i}{-4+16i}$	
4	Evaluate: $\left\|-65+72i\right\|$	
5	Determine if the following statement is always, sometimes, or never true (you may just write "always", "sometimes", or "never" as your answer): "The sum of two real numbers is a complex number."	

	5 points each	
6	What is the sum of the magnitudes of the eight complex solutions of the equation $x^8=1$?	
7	Simplify: $\frac{\sqrt{-5}\cdot\sqrt{-15}}{\sqrt{(-5)(-15)}}$	
8	For the polynomial $f(x)=x^3+12x^2+bx+60$, where b is real, one of the roots equals the sum of the other two roots. Find the complex root of f whose imaginary part is positive.	
9	Find the rectangular form of the complex number with polar form $-2\text{cis}\left(-\frac{\pi}{3}\right)$.	
10	Write in $a+bi$ form: $\frac{(1+i)^{17}}{(1-i)^{16}}$	

	6 points each	
11	Find the sum of all positive integers $n<100$ such that $\left(\sqrt{3}-i\right)^n$ is a real number.	
12	Consider the set $\{1,-1,i,-i\}$. The reciprocal of one element of the set equals the conjugate of a different element of the set for how many elements in the set?	
13	Find all complex solutions to the equation $ix^2+(1-5i)x-1+8i=0$.	
14	Write in $a+bi$ form: $\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{6}-\sqrt{2}}{4}i\right)^{2014}$	
15	Consider polynomial $f(x)=x^5+ax^4+bx^3+cx^2+dx-442$ with integral coefficients. f has one real root and four imaginary roots. The product of two of the imaginary roots is $11+10i$. Find the value of the real root of f.	

2013 – 2014 Log1 Contest Round 2
Mu Complex Numbers

Name: ________________

4 points each		
1	Write in $a+bi$ form: $(-1+i)+(3+5i)-(2-i)$	
2	Write in $a+bi$ form: $(-1+i)(3+5i)(2-i)$	
3	Write in $a+bi$ form: $\frac{6+5i}{-4+16i}$	
4	Evaluate: $\lvert -65+72i \rvert$	
5	Using Euler's Formula, find the value of $\cos(i)-i\sin(i)$, where *i* is the imaginary unit.	

5 points each		
6	What is the sum of the magnitudes of the eight complex solutions of the equation $x^8=1$?	
7	Simplify: $\frac{\sqrt{-5}\cdot\sqrt{-15}}{\sqrt{(-5)(-15)}}$	
8	For the polynomial $f(x)=x^3+12x^2+bx+60$, where *b* is real, one of the roots equals the sum of the other two roots. Find the complex root of *f* whose imaginary part is positive.	
9	Find the rectangular form of the complex number with polar form $-2\,\mathrm{cis}\left(-\frac{\pi}{3}\right)$.	
10	Write in $a+bi$ form: $\frac{(1+i)^{17}}{(1-i)^{16}}$	

6 points each		
11	Find the sum of all positive integers $n<100$ such that $\left(\sqrt{3}-i\right)^n$ is a real number.	
12	Find all complex solutions to the equation $ix^2+\left(1-5i\right)x-1+8i=0$.	
13	Consider the complex numbers that are solutions to the equation $x^n=1$, where n is a positive integer satisfying $n\geq 3$. If the solutions are plotted in the Argand plane, and if those solutions are the vertices of a regular polygon whose enclosed area is A_n, find the value of $\lim_{n\to\infty} A_n$.	
14	Write in $a+bi$ form: $\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{6}-\sqrt{2}}{4}i\right)^{2014}$	
15	Consider polynomial $f\left(x\right)=x^5+ax^4+bx^3+cx^2+dx-442$ with integral coefficients. f has one real root and four imaginary roots. The product of two of the imaginary roots is $11+10i$. Find the value of the real root of f.	

2013 – 2014 Log1 Contest Round 2
Theta Complex Numbers

Name: ________________

	4 points each	
1	Write in $a+bi$ form: $(-1+i)+(3+5i)-(2-i)$	$7i$
2	Write in $a+bi$ form: $(-1+i)(3+5i)(2-i)$	$-18+4i$
3	Write in $a+bi$ form: $\dfrac{6+5i}{-4+16i}$	$\dfrac{7}{34}-\dfrac{29}{68}i$
4	Evaluate: $\lvert -65+72i \rvert$	97
5	Determine if the following statement is always, sometimes, or never true (you may just write "always", "sometimes", or "never" as your answer): "The sum of two real numbers is a complex number."	always

	5 points each	
6	What is the sum of the magnitudes of the eight complex solutions of the equation $x^8=1$?	8
7	Simplify: $\dfrac{\sqrt{-5}\cdot\sqrt{-15}}{\sqrt{(-5)(-15)}}$	-1
8	For the polynomial $f(x)=x^3+12x^2+bx+60$, where b is real, one of the roots equals the sum of the other two roots. Find the complex root of f whose imaginary part is positive.	$-3+i$
9	For complex number $z=2-3i$, find the value of $z\cdot\bar{z}$.	13
10	Find the distance between the complex numbers $-2+i$ and $3+5i$ in the Argand plane.	$\sqrt{41}$

	6 points each	
11	Write in $a+bi$ form: $\dfrac{(1+i)^{17}}{(1-i)^{16}}$	$1+i$
12	Find the sum of all positive integers $n<100$ such that $\left(\sqrt{3}-i\right)^n$ is a real number.	816
13	Find all complex solutions to the equation $ix^2+(1-5i)x-1+8i=0$.	$2-i,\ 3+2i$
14	Write in $a+bi$ form: $\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}+\dfrac{\sqrt{6}-\sqrt{2}}{4}i\right)^{2014}$	$\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i$
15	Consider polynomial $f(x)=x^5+ax^4+bx^3+cx^2+dx-442$ with integral coefficients. f has one real root and four imaginary roots. The product of two of the imaginary roots is $11+10i$. Find the value of the real root of f.	2

2013 – 2014 Log1 Contest Round 2
Alpha Complex Numbers

Name: ________________

	4 points each	
1	Write in $a+bi$ form: $(-1+i)+(3+5i)-(2-i)$	$7i$
2	Write in $a+bi$ form: $(-1+i)(3+5i)(2-i)$	$-18+4i$
3	Write in $a+bi$ form: $\frac{6+5i}{-4+16i}$	$\frac{7}{34}-\frac{29}{68}i$
4	Evaluate: $\lvert -65+72i \rvert$	97
5	Determine if the following statement is always, sometimes, or never true (you may just write "always", "sometimes", or "never" as your answer): "The sum of two real numbers is a complex number."	always

	5 points each	
6	What is the sum of the magnitudes of the eight complex solutions of the equation $x^8=1$?	8
7	Simplify: $\frac{\sqrt{-5}\cdot\sqrt{-15}}{\sqrt{(-5)(-15)}}$	-1
8	For the polynomial $f(x)=x^3+12x^2+bx+60$, where b is real, one of the roots equals the sum of the other two roots. Find the complex root of f whose imaginary part is positive.	$-3+i$
9	Find the rectangular form of the complex number with polar form $-2\operatorname{cis}\left(-\frac{\pi}{3}\right)$.	$-1+\sqrt{3}i$
10	Write in $a+bi$ form: $\frac{(1+i)^{17}}{(1-i)^{16}}$	$1+i$

	6 points each	
11	Find the sum of all positive integers $n<100$ such that $\left(\sqrt{3}-i\right)^n$ is a real number.	816
12	Consider the set $\{1,-1,i,-i\}$. The reciprocal of one element of the set equals the conjugate of a different element of the set for how many elements in the set?	0
13	Find all complex solutions to the equation $ix^2+(1-5i)x-1+8i=0$.	$2-i,\ 3+2i$
14	Write in $a+bi$ form: $\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{6}-\sqrt{2}}{4}i\right)^{2014}$	$\frac{\sqrt{3}}{2}-\frac{1}{2}i$
15	Consider polynomial $f(x)=x^5+ax^4+bx^3+cx^2+dx-442$ with integral coefficients. f has one real root and four imaginary roots. The product of two of the imaginary roots is $11+10i$. Find the value of the real root of f.	2

2013 – 2014 Log1 Contest Round 2
Mu Complex Numbers

Name: ________________

4 points each		
1	Write in $a+bi$ form: $(-1+i)+(3+5i)-(2-i)$	$7i$
2	Write in $a+bi$ form: $(-1+i)(3+5i)(2-i)$	$-18+4i$
3	Write in $a+bi$ form: $\dfrac{6+5i}{-4+16i}$	$\dfrac{7}{34}-\dfrac{29}{68}i$
4	Evaluate: $\lvert -65+72i \rvert$	97
5	Using Euler's Formula, find the value of $\cos(i)-i\sin(i)$, where *i* is the imaginary unit.	e

5 points each		
6	What is the sum of the magnitudes of the eight complex solutions of the equation $x^8=1$?	8
7	Simplify: $\dfrac{\sqrt{-5}\cdot\sqrt{-15}}{\sqrt{(-5)(-15)}}$	-1
8	For the polynomial $f(x)=x^3+12x^2+bx+60$, where *b* is real, one of the roots equals the sum of the other two roots. Find the complex root of *f* whose imaginary part is positive.	$-3+i$
9	Find the rectangular form of the complex number with polar form $-2\,\mathrm{cis}\left(-\dfrac{\pi}{3}\right)$.	$-1+\sqrt{3}i$
10	Write in $a+bi$ form: $\dfrac{(1+i)^{17}}{(1-i)^{16}}$	$1+i$

6 points each		
11	Find the sum of all positive integers $n<100$ such that $\left(\sqrt{3}-i\right)^{n}$ is a real number.	816
12	Find all complex solutions to the equation $ix^2+\left(1-5i\right)x-1+8i=0$.	$2-i,\ 3+2i$
13	Consider the complex numbers that are solutions to the equation $x^n=1$, where n is a positive integer satisfying $n\geq 3$. If the solutions are plotted in the Argand plane, and if those solutions are the vertices of a regular polygon whose enclosed area is A_n, find the value of $\lim_{n\to\infty} A_n$.	π
14	Write in $a+bi$ form: $\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}+\dfrac{\sqrt{6}-\sqrt{2}}{4}i\right)^{2014}$	$\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i$
15	Consider polynomial $f\left(x\right)=x^5+ax^4+bx^3+cx^2+dx-442$ with integral coefficients. f has one real root and four imaginary roots. The product of two of the imaginary roots is $11+10i$. Find the value of the real root of f.	2

2013 – 2014 Log1 Contest Round 2
Complex Numbers Solutions

Mu	Al	Th	Solution
1	1	1	$(-1+i)+(3+5i)-(2-i)=(-1+3-2)+(1+5-(-1))i=7i$
2	2	2	$(-1+i)(3+5i)(2-i)=(-8-2i)(2-i)=-18+4i$
3	3	3	$\frac{6+5i}{-4+16i}=\frac{6+5i}{-4+16i}\cdot\frac{-4-16i}{-4-16i}=\frac{-24-96i-20i+80}{16+256}=\frac{56-116i}{272}=\frac{7}{34}-\frac{29}{68}i$ $\left(\frac{14-29i}{68}\right.$ or comparable answer is not in $a+bi$ form$\left.\right)$
4	4	4	$\lvert -65+72i\rvert=\sqrt{(-65)^2+72^2}=\sqrt{4225+5184}=\sqrt{9409}=97$
	5	5	Since a complex number is a number of the form $a+bi$, where *a* and *b* are real, all real numbers are complex numbers (just make $b=0$). Since two real numbers' sum is always real, their sum is also always complex.
5			$\cos(i)-i\sin(i)=\cos(-i)+i\sin(-i)=e^{i(-i)}=e^1=e$
6	6	6	The solutions to $x^8=1$ are all eighth roots of unity, so each solution's magnitude is 1. Therefore, since there are eight distinct solutions, the sum of the eight solutions' magnitudes is 8.
7	7	7	$\frac{\sqrt{-5}\cdot\sqrt{-15}}{\sqrt{(-5)(-15)}}=\frac{i\sqrt{5}\cdot i\sqrt{15}}{\sqrt{75}}=i^2=-1$
8	8	8	Because the sum of all roots is $-\frac{12}{1}=-12$, and because one of the roots equals the sum of the other two, one of the roots must be -6 while the other two sum to -6. Therefore, $0=(-6)^3+12(-6)^2-6b+60=-6b+276\Rightarrow b=46$. Thus, x^3+12x^2+46x $+60=(x+6)(x^2+6x+10)$, and the other two roots are $\frac{-6\pm\sqrt{6^2-4\cdot1\cdot10}}{2\cdot1}=-3\pm i$. Of the three roots, the only one with positive imaginary part is $-3+i$.
		9	$z\cdot\overline{z}=(2-3i)(2+3i)=4-9i^2=4+9=13$
9	9		$-2cis\left(-\frac{\pi}{3}\right)=-2\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)=-2\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=-1+\sqrt{3}i$
		10	$\sqrt{(-2-3)^2+(1-5)^2}=\sqrt{(-5)^2+(-4)^2}=\sqrt{25+16}=\sqrt{41}$
10	10	11	Since $\frac{1+i}{1-i}=i$, $\frac{(1+i)^{17}}{(1-i)^{16}}=\left(\frac{1+i}{1-i}\right)^{16}(1+i)=i^{16}(1+i)=1(1+i)=1+i$.
11	11	12	$\left(\sqrt{3}-i\right)^n=\left(2cis(-30^\circ)\right)^n=2^n cis(-30^\circ n)$, and this will be real if $-30^\circ n$ is coterminal with 0° or 180°. Therefore, *n* must be a multiple of 6, and the desired sum is $6+12+18+...+96=\frac{16}{2}(6+96)=8\cdot102=816$.

	12		In $\{1,-1,i,-i\}$, the reciprocals of the elements are 1, -1, $-i$, and i, respectively, and the conjugates of these elements are the original elements in the set, respectively. Therefore, no element equals the conjugate of a different element. In actuality, each element's reciprocal is the conjugate of the exact same element.
12	13	13	Using the quadratic formula, $x=\frac{-(1-5i)\pm\sqrt{(1-5i)^2-4i(-1+8i)}}{2i}=\frac{5i-1\pm\sqrt{8-6i}}{2i}$ $=\frac{5i-1\pm(3-i)}{2i}$, and the two solutions are $2-i$ and $3+2i$.
13			The vertices are equally spread on the unit circle, so as $n\to\infty$, the polygon approaches the unit circle, the enclosed area of which is $\pi(1)^2=\pi$.
14	14	14	$\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{6}-\sqrt{2}}{4}i\right)^{2014}=\left(cis\left(15^\circ\right)\right)^{2014}=cis\left(2014\cdot 15^\circ\right)=cis30210^\circ=cis330^\circ$ $=\frac{\sqrt{3}}{2}-\frac{1}{2}i$
15	15	15	Since the coefficients of f are integers, the product of the other two roots must be $11-10i$, making the product of all four imaginary roots $(11+10i)(11-10i)=11^2+10^2$ $=221$. Since the product of all five roots is $-\frac{-442}{1}=442$, the real root must be 2.