2013 – 2014 Log1 Contest Round 1
Theta Equations & Inequalities

Name: _____

4 points each		
1	Solve for x: $-5x+3=38$	
2	Find the greatest integral value of x satisfying the inequality $(-3x+7)(x+1)>3$.	
3	Find the ordered pair solution to the system of equations $\begin{cases} 4x+5y=71 \\ 6x+7y=109 \end{cases}$.	
4	How many real solutions does the equation $\ln x=\frac{1}{x}$ have?	
5	Find the sum of the solutions of the equation $4x^3-20x^2+100x-15=0$.	

5 points each		
6	Solve for x: $\log_2(x^2+3)-\log_2(x+1)=4$	
7	Given the equations $a+b+c=6$, $a+b+d=7$, $a+c+d=8$, and $b+c+d=9$, find the value of the product $abcd$.	
8	Find the area of the region defined by the system $\begin{cases} y\le 8x-17 \\ y\ge x-3 \\ 5x+2y\le 29 \end{cases}$.	
9	Within 20 years, in what year is the equal sign (=) generally attributed as being first used in print to represent equality?	
10	Given the equations $4x+6y-5z=42$, $5x-3y+z=8$, and $-7x+2y+6z=3$, find the value of $-15x+13y+6z$.	

6 points each		
11	Find the sum of the squares of the solutions of the equation $x^5-9x^4+6x^3+2x^2+x-9=0$.	
12	If $\sin x-\cos x=1$, find the value of $\sin\left(x-\frac{\pi}{4}\right)$.	
13	A right rectangular prism measures $x\times y\times z$, with all lengths measured in cm. If the space diagonal of the prism has length 22 cm, and if $x+y+z=26$ cm, find the total surface area of the prism, in cm^2.	
14	An ellipse has minor axis of length 4, and the eccentricity of the ellipse is numerically equal to the length of the ellipse's latus rectum. Find the ellipse's major axis length.	
15	Find the oblique asymptote of the graph with equation $y=\frac{4x^4-3x^3+17x^2+x-24}{2x^3+3x^2+x-14}$, written in slope-intercept form.	

2013 – 2014 Log1 Contest Round 1
Alpha Equations & Inequalities

Name: _____

	4 points each	
1	Solve for x: $-5x+3=38$	
2	Find the greatest integral value of x satisfying the inequality $(-3x+7)(x+1)>3$.	
3	Find the ordered pair solution to the system of equations $\begin{cases} 4x+5y=71 \\ 6x+7y=109 \end{cases}$.	
4	Find the sum of the solutions of the equation $4x^3-20x^2+100x-15=0$.	
5	Solve for x: $\log_2(x^2+3)-\log_2(x+1)=4$	

	5 points each	
6	Given the equations $a+b+c=6$, $a+b+d=7$, $a+c+d=8$, and $b+c+d=9$, find the value of the product $abcd$.	
7	The ellipse with equation $7x^2-6\sqrt{3}xy+13y^2=16$ is generated by rotating counterclockwise an ellipse with horizontal major axis. Find the least positive radian measure angle θ by which this could be achieved.	
8	Find the area of the region defined by the system $\begin{cases} y\le 8x-17 \\ y\ge x-3 \\ 5x+2y\le 29 \end{cases}$.	
9	Within 20 years, in what year is the equal sign (=) generally attributed as being first used in print to represent equality?	
10	Find the sum of the squares of the solutions of the equation $x^5-9x^4+6x^3+2x^2+x-9=0$.	

	6 points each	
11	If $\sin x-\cos x=1$, find the value of $\sin\left(x-\frac{\pi}{4}\right)$.	
12	A right rectangular prism measures $x\times y\times z$, with all lengths measured in cm. If the space diagonal of the prism has length 22 cm, and if $x+y+z=26$ cm, find the total surface area of the prism, in cm^2.	
13	How many ordered quadruples of non-negative integers (x,y,z,w) satisfy the equation $x+y+z+w=12$?	
14	An ellipse has minor axis of length 4, and the eccentricity of the ellipse is numerically equal to the length of the ellipse's latus rectum. Find the ellipse's major axis length.	
15	Find the oblique asymptote of the graph with equation $y=\frac{4x^4-3x^3+17x^2+x-24}{2x^3+3x^2+x-14}$, written in slope-intercept form.	

2013 – 2014 Log1 Contest Round 1
Mu Equations & Inequalities

Name: _____

	4 points each	
1	Solve for x: $-5x+3=38$	
2	Find the greatest integral value of x satisfying the inequality $(-3x+7)(x+1)>3$.	
3	Find the sum of the solutions of the equation $4x^3-20x^2+100x-15=0$.	
4	Find the numerical value of $\left.\frac{dy}{dx}\right\|_{(x,y)=(3,-2)}$ when $x^2+xy-4y=11$.	
5	The ellipse with equation $7x^2-6\sqrt{3}xy+13y^2=16$ is generated by rotating counterclockwise an ellipse with horizontal major axis. Find the least positive radian measure angle θ by which this could be achieved.	

	5 points each	
6	Find the area of the region defined by the system $\begin{cases} y\le 8x-17 \\ y\ge x-3 \\ 5x+2y\le 29 \end{cases}$.	
7	$\lim_{n\to\infty}\left(\frac{2}{n}\sum_{i=1}^{n}\left(2+\sin\left(1+\frac{2i}{n}\right)\right)\right)$ is the area enclosed by $y=2+\sin x$, the x-axis, and what two vertical lines?	
8	Let A, B, C, and D be the solutions of $x^4-x^3-46x^2+16x+480=0$, where $A<B<C<D$. Find the value of $\left(\frac{B+C}{A+D}\right)^5$.	
9	Within 20 years, in what year is the equal sign (=) generally attributed as being first used in print to represent equality?	
10	Find the sum of the squares of the solutions of the equation $x^5-9x^4+6x^3+2x^2+x-9=0$.	

6 points each		
11	What is the greatest real value of k such that $\sin x - \cos x = k$ has a real solution?	
12	A right rectangular prism measures $x \times y \times z$, with all lengths measured in cm. If the space diagonal of the prism has length 22 cm, and if $x + y + z = 26$ cm, find the total surface area of the prism, in cm^2.	
13	How many ordered quadruples of non-negative integers (x, y, z, w) satisfy the equation $x + y + z + w = 12$?	
14	An ellipse has minor axis of length 4, and the eccentricity of the ellipse is numerically equal to the length of the ellipse's latus rectum. Find the ellipse's major axis length.	
15	Find the solution to the equation $\frac{dy}{dx} = 2xy$ through the point $\left(2, 2e^4\right)$, written in the form $y = f(x)$.	

2013 – 2014 Log1 Contest Round 1
Theta Equations & Inequalities

Name: _____

4 points each		
1	Solve for x: $-5x+3=38$	-7
2	Find the greatest integral value of x satisfying the inequality $(-3x+7)(x+1)>3$.	1
3	Find the ordered pair solution to the system of equations $\begin{cases} 4x+5y=71 \\ 6x+7y=109 \end{cases}$.	$(24,-5)$
4	How many real solutions does the equation $\ln x=\frac{1}{x}$ have?	1
5	Find the sum of the solutions of the equation $4x^3-20x^2+100x-15=0$.	5

5 points each		
6	Solve for x: $\log_2(x^2+3)-\log_2(x+1)=4$	$8\pm\sqrt{77}$
7	Given the equations $a+b+c=6$, $a+b+d=7$, $a+c+d=8$, and $b+c+d=9$, find the value of the product $abcd$.	24
8	Find the area of the region defined by the system $\begin{cases} y\le 8x-17 \\ y\ge x-3 \\ 5x+2y\le 29 \end{cases}$.	$\frac{21}{2}$
9	Within 20 years, in what year is the equal sign (=) generally attributed as being first used in print to represent equality?	1557 (accept any year from 1537 to 1577)
10	Given the equations $4x+6y-5z=42$, $5x-3y+z=8$, and $-7x+2y+6z=3$, find the value of $-15x+13y+6z$.	40

6 points each		
11	Find the sum of the squares of the solutions of the equation $x^5-9x^4+6x^3+2x^2+x-9=0$.	69
12	If $\sin x-\cos x=1$, find the value of $\sin\left(x-\frac{\pi}{4}\right)$.	$\frac{\sqrt{2}}{2}$
13	A right rectangular prism measures $x\times y\times z$, with all lengths measured in cm. If the space diagonal of the prism has length 22 cm, and if $x+y+z=26$ cm, find the total surface area of the prism, in cm^2.	192
14	An ellipse has minor axis of length 4, and the eccentricity of the ellipse is numerically equal to the length of the ellipse's latus rectum. Find the ellipse's major axis length.	$4\sqrt{17}$
15	Find the oblique asymptote of the graph with equation $y=\frac{4x^4-3x^3+17x^2+x-24}{2x^3+3x^2+x-14}$, written in slope-intercept form.	$y=2x-\frac{9}{2}$

2013 – 2014 Log1 Contest Round 1
Alpha Equations & Inequalities

Name: _____

4 points each		
1	Solve for x: $-5x+3=38$	-7
2	Find the greatest integral value of x satisfying the inequality $(-3x+7)(x+1)>3$.	1
3	Find the ordered pair solution to the system of equations $\begin{cases} 4x+5y=71 \\ 6x+7y=109 \end{cases}$.	$(24,-5)$
4	Find the sum of the solutions of the equation $4x^3-20x^2+100x-15=0$.	5
5	Solve for x: $\log_2(x^2+3)-\log_2(x+1)=4$	$8\pm\sqrt{77}$

5 points each		
6	Given the equations $a+b+c=6$, $a+b+d=7$, $a+c+d=8$, and $b+c+d=9$, find the value of the product $abcd$.	24
7	The ellipse with equation $7x^2-6\sqrt{3}xy+13y^2=16$ is generated by rotating counterclockwise an ellipse with horizontal major axis. Find the least positive radian measure angle θ by which this could be achieved.	$\frac{\pi}{6}$
8	Find the area of the region defined by the system $\begin{cases} y\le 8x-17 \\ y\ge x-3 \\ 5x+2y\le 29 \end{cases}$.	$\frac{21}{2}$
9	Within 20 years, in what year is the equal sign (=) generally attributed as being first used in print to represent equality?	1557 (accept any year from 1537 to 1577)
10	Find the sum of the squares of the solutions of the equation $x^5-9x^4+6x^3+2x^2+x-9=0$.	69

6 points each		
11	If $\sin x - \cos x = 1$, find the value of $\sin\left(x - \frac{\pi}{4}\right)$.	$\frac{\sqrt{2}}{2}$
12	A right rectangular prism measures $x \times y \times z$, with all lengths measured in cm. If the space diagonal of the prism has length 22 cm, and if $x + y + z = 26$ cm, find the total surface area of the prism, in cm^2.	192
13	How many ordered quadruples of non-negative integers (x, y, z, w) satisfy the equation $x + y + z + w = 12$?	455
14	An ellipse has minor axis of length 4, and the eccentricity of the ellipse is numerically equal to the length of the ellipse's latus rectum. Find the ellipse's major axis length.	$4\sqrt{17}$
15	Find the oblique asymptote of the graph with equation $y = \frac{4x^4 - 3x^3 + 17x^2 + x - 24}{2x^3 + 3x^2 + x - 14}$, written in slope-intercept form.	$y = 2x - \frac{9}{2}$

2013 – 2014 Log1 Contest Round 1
Mu Equations & Inequalities

Name: _____

4 points each		
1	Solve for x: $-5x+3=38$	-7
2	Find the greatest integral value of x satisfying the inequality $(-3x+7)(x+1)>3$.	1
3	Find the sum of the solutions of the equation $4x^3-20x^2+100x-15=0$.	5
4	Find the numerical value of $\left.\frac{dy}{dx}\right\|_{(x,y)=(3,-2)}$ when $x^2+xy-4y=11$.	4
5	The ellipse with equation $7x^2-6\sqrt{3}xy+13y^2=16$ is generated by rotating counterclockwise an ellipse with horizontal major axis. Find the least positive radian measure angle θ by which this could be achieved.	$\frac{\pi}{6}$

5 points each		
6	Find the area of the region defined by the system $\begin{cases} y\le 8x-17 \\ y\ge x-3 \\ 5x+2y\le 29 \end{cases}$.	$\frac{21}{2}$
7	$\lim_{n\to\infty}\left(\frac{2}{n}\sum_{i=1}^{n}\left(2+\sin\left(1+\frac{2i}{n}\right)\right)\right)$ is the area enclosed by $y=2+\sin x$, the x-axis, and what two vertical lines?	$x=1$, $x=3$
8	Let A, B, C, and D be the solutions of $x^4-x^3-46x^2+16x+480=0$, where $A<B<C<D$. Find the value of $\left(\frac{B+C}{A+D}\right)^5$.	0
9	Within 20 years, in what year is the equal sign (=) generally attributed as being first used in print to represent equality?	1557 (accept any year from 1537 to 1577)
10	Find the sum of the squares of the solutions of the equation $x^5-9x^4+6x^3+2x^2+x-9=0$.	69

6 points each		
11	What is the greatest real value of k such that $\sin x - \cos x = k$ has a real solution?	$\sqrt{2}$
12	A right rectangular prism measures $x \times y \times z$, with all lengths measured in cm. If the space diagonal of the prism has length 22 cm, and if $x + y + z = 26$ cm, find the total surface area of the prism, in cm^2.	192
13	How many ordered quadruples of non-negative integers (x, y, z, w) satisfy the equation $x + y + z + w = 12$?	455
14	An ellipse has minor axis of length 4, and the eccentricity of the ellipse is numerically equal to the length of the ellipse's latus rectum. Find the ellipse's major axis length.	$4\sqrt{17}$
15	Find the solution to the equation $\frac{dy}{dx} = 2xy$ through the point $(2, 2e^4)$, written in the form $y = f(x)$.	$y = 2e^{x^2}$

2013 – 2014 Log1 Contest Round 1
Equations & Inequalities Solutions

Mu	Al	Th	Solution
1	1	1	$-5x+3=38\Rightarrow -5x=35\Rightarrow x=-7$
2	2	2	$3<(-3x+7)(x+1)=-3x^2+4x+7\Rightarrow 0>3x^2-4x-4=(3x+2)(x-2)\Rightarrow -\frac{2}{3}<x<2$. Therefore, the greatest integral value of x satisfying the inequality is 1.
	3	3	Subtracting the first equation from the second yields $2x+2y=38\Rightarrow 4x+4y=76$. Subtracting this second equation from the first original equation yields $y=-5$. Plugging this in to any equation yields $x=24$, so the solution is $(24,-5)$.
		4	Examining the graphs of $y=\ln x$ and $y=\frac{1}{x}$, the first exists only in quadrants I and IV while the second exists only in quadrants I and III, so quadrant I is where an intersection point could exist. Secondly, the first graph increases without bound while the second graph decreases asymptotically toward the *x*-axis, so there can only be one intersection point.
3	4	5	The sum of the solutions is $-\frac{-20}{4}=5$.
4			Differentiating implicitly, $2x+x\frac{dy}{dx}+y-4\frac{dy}{dx}=0$, and plugging in the point $(3,-2)$ yields $6+3\left.\frac{dy}{dx}\right\|_{(x,y)=(3,-2)}-2-4\left.\frac{dy}{dx}\right\|_{(x,y)=(3,-2)}=0\Rightarrow \left.\frac{dy}{dx}\right\|_{(x,y)=(3,-2)}=4$.
	5	6	$4=\log_2\left(\frac{x^2+3}{x+1}\right)\Rightarrow \frac{x^2+3}{x+1}=2^4=16\Rightarrow x^2+3=16x+16\Rightarrow 0=x^2-16x-13$. Using the quadratic formula, $x=\frac{-(-16)\pm\sqrt{(-16)^2-4(1)(-13)}}{2(1)}=8\pm\sqrt{77}$. Since both values are greater than -1, both logarithms are defined, and thus both are solutions.
	6	7	Summing the four equations yields $3(a+b+c+d)=30\Rightarrow a+b+c+d=10$. Subtracting each of the given equations from this last equation yields $d=4$, $c=3$, $b=2$, and $a=1$. Therefore, the product is $abcd=1\cdot 2\cdot 3\cdot 4=24$.
5	7		$\tan 2\theta=\frac{-6\sqrt{3}}{7-13}=\sqrt{3}\Rightarrow 2\theta=\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{6}$
6	8	8	The region is a triangular region with vertices at the points $(2,-1)$, $(3,7)$, and $(5,2)$. Using the shoelace method, the enclosed area is $\begin{vmatrix} 3 & 7 \\ 5 & 2 \\ 2 & -1 \\ 3 & 7 \end{vmatrix}\begin{matrix} \\ 6 \\ -5 \\ 14 \end{matrix}\begin{matrix} \\ 35 \\ 4 \\ -3 \\ 36 \end{matrix}\begin{matrix} \\ \\ \Rightarrow A=\frac{1}{2}\lvert 36-15\rvert=\frac{21}{2} \\ \\ \\ 15 \end{matrix}$.

7			Since $\int_a^b f(x)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\sum_{i=1}^{n}\left(f\left(a+i\frac{b-a}{n}\right)\right)\right)$, $\lim_{n\to\infty}\left(\frac{2}{n}\sum_{i=1}^{n}\left(2+\sin\left(1+\frac{2i}{n}\right)\right)\right)$ represents the area enclosed by $y=2+\sin x$ (since $f>0$ always), the x-axis, $x=a$, and $x=b$, where $a=1$ and $b-a=2\Rightarrow b=3$. Therefore, the two vertical lines are $x=1$ and $x=3$.
8			$0=x^4-x^3-46x^2+16x+480=(x+5)(x+4)(x-4)(x-6)$, so $A=-5$, $B=-4$, $C=4$, and $D=6$. Therefore, $\left(\frac{B+C}{A+D}\right)^5=\left(\frac{-4+4}{-5+6}\right)^5=0^5=0$.
9	9	9	Robert Recorde is generally attributed with using the symbol = to represent equals in his book *The Whetstone of Witte* in 1557.
		10	If you take the first equation, subtract the second equation from that, then add twice the third equation, the result is $-15x+13y+6z=40$. Alternately, the solution of the system is $(5,7,4)$, and $-15x+13y+6z=-15\cdot5+13\cdot7+6\cdot4=-75+91+24=40$.
10	10	11	Using the Newton's sums method, the sum of the solutions is $S_1=-\frac{-9}{1}=9$, and the sum of the squares of the solutions S_2 satisfies $S_2-9S_1+6\cdot2=0\Rightarrow S_2=9\cdot9-12=69$.
	11	12	$\sin\left(x-\frac{\pi}{4}\right)=\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}(\sin x-\cos x)=\frac{\sqrt{2}}{2}\cdot1=\frac{\sqrt{2}}{2}$
11			This is the same as asking what the maximum value is of the function $f(x)=\sin x-\cos x$. $f'(x)=\cos x+\sin x$ and $f'(x)=0$ if $\cos x=-\sin x$. Therefore, the maximum occurs when $\cos x=-\frac{\sqrt{2}}{2}$ and $\sin x=\frac{\sqrt{2}}{2}$, making the maximum value of f $\frac{\sqrt{2}}{2}-\left(-\frac{\sqrt{2}}{2}\right)=\sqrt{2}$.
12	12	13	Since the diagonal has length $\sqrt{x^2+y^2+z^2}$, $484=22^2=x^2+y^2+z^2$. Therefore, $676=26^2=(x+y+z)^2=x^2+y^2+z^2+2xy+2xz+2yz=484+2xy+2xz+2yz$ $\Rightarrow 2xy+2xz+2yz=192$, and $2xy+2xz+2yz$ is the surface area.
13	13		The total number of non-negative integral quadruples is $\binom{4+12-1}{12}=\binom{15}{12}=455$.
14	14	14	Let a and b be the semi-major and semi-minor axis lengths, respectively, and let c be the distance from either focus to the center of the ellipse. Since the latus rectum length equals the eccentricity, $\frac{2b^2}{a}=\frac{c}{a}\Rightarrow 2b^2=c$. Additionally, since this is an ellipse, $a^2-b^2=c^2=\left(2b^2\right)^2=4b^4$. Therefore, $a^2=4b^4+b^2\Rightarrow a=\sqrt{4b^4+b^2}=b\sqrt{4b^2+1}$. Therefore, $a=2\sqrt{4(2)^2+1}=2\sqrt{17}$, making the major axis length $4\sqrt{17}$.
	15	15	$\frac{4x^4-3x^3+17x^2+x-24}{2x^3+3x^2+x-14}=2x-\frac{9}{2}+\frac{\frac{57}{2}x^2+\frac{67}{2}x-87}{2x^3+3x^2+x-14}$, so the oblique asymptote is $y=2x-\frac{9}{2}$.

15			$\int \frac{dy}{y} = \int 2x dx \Rightarrow \ln\lvert y\rvert = x^2 + c \Rightarrow y = Ae^{x^2}$. Since the graph passes through the point $\left(2, 2e^4\right)$, $2e^4 = Ae^{2^2} = Ae^4 \Rightarrow A = 2$. Therefore, the solution is $y = 2e^{x^2}$.