2012 – 2013 Log1 Contest Round 1
Theta Matrices

Name: ________________

4 points each		
1	Evaluate: $2\begin{bmatrix}1 & -3\\2 & 4\end{bmatrix}-3\begin{bmatrix}2 & -1\\-1 & -4\end{bmatrix}$	
2	Evaluate: $\begin{vmatrix}-4 & 5\\-5 & 6\end{vmatrix}$	
3	Solve for x : $\begin{bmatrix}x & 2 & 3\\7 & 6 & 8\end{bmatrix}\begin{bmatrix}5\\3\\1\end{bmatrix}=\begin{bmatrix}19\\61\end{bmatrix}$	
4	Solve for x : $\begin{vmatrix}x & -6 & 2\\7 & 1 & 0\\1 & 3 & 5\end{vmatrix}=250$	
5	If A is a 2×2 matrix whose inverse is equal to A, find A^{2012}.	

5 points each		
6	How much area is enclosed by a triangle whose vertices are at the points $(7,3)$, $(-1,-5)$, and $(3,-3)$?	
7	Find the trace of the matrix $\begin{bmatrix}1 & 5 & 9 & 13\\2 & 6 & 10 & 14\\3 & 7 & 11 & 15\\4 & 8 & 12 & 16\end{bmatrix}$.	
8	Find the inverse of $\begin{bmatrix}-2 & 12\\0 & 4\end{bmatrix}$, with entries written as decimals.	
9	How many entries in the product $\begin{bmatrix}-3 & 7\\-2 & 0\\6 & -2\end{bmatrix}\begin{bmatrix}0 & 4 & 4 & -3 & 1\\1 & -2 & 0 & 2 & 1\end{bmatrix}$ are not equal to 0?	
10	When using Cramer's Rule on the system $\begin{cases}2x+3y=117\\-2x+5y=40\end{cases}$, the solution for x can be written as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find the product ab.	

6 points each		
11	Find the ordered pair solution to the system $\begin{cases} 14x-15y=10 \\ -15x+16y=-11 \end{cases}$.	
12	Find the element in the third row and second column for the inverse of the matrix $\begin{bmatrix} 2 & 3 & 5 \\ 4 & -6 & 1 \\ -1 & 4 & 1 \end{bmatrix}$.	
13	A square matrix A_n whose dimensions are $n \times n$ is defined by having each entry equal to the sum of its row and column numbers. For example, the element in the third row and second column of A_6 is $3+2=5$. Consider the determinants of A_2, A_3, A_4, ..., A_n, ..., where $n \geq 2$. Find the minimum value of all such determinants.	
14	If A, B, C, and D have dimensions of 2×6, 3×4, 2×3, and 6×4, respectively, which of the following is not a square matrix: $AD(CB)^T$, $(CB)^T AD$, $B^T B(AD)^T$, or $\left(A^T ADD^T\right)^T$?	
15	What value of x makes the matrix $\begin{bmatrix} 4 & -2 & 1 \\ 3 & x & -6 \\ 2 & 5 & -3 \end{bmatrix}$ singular?	

2012 – 2013 Log1 Contest Round 1
Alpha Matrices

Name: ________________

	4 points each	
1	Evaluate: $2\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} - 3\begin{bmatrix} 2 & -1 \\ -1 & -4 \end{bmatrix}$	
2	Evaluate: $\begin{vmatrix} -4 & 5 \\ -5 & 6 \end{vmatrix}$	
3	Solve for x: $\begin{bmatrix} x & 2 & 3 \\ 7 & 6 & 8 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 61 \end{bmatrix}$	
4	Solve for x: $\begin{vmatrix} x & -6 & 2 \\ 7 & 1 & 0 \\ 1 & 3 & 5 \end{vmatrix} = 250$	
5	A matrix A is called idempotent if $A^2 = A$. If A is a 3×3 idempotent invertible matrix, find A.	

	5 points each	
6	How much area is enclosed by a triangle whose vertices are at the points $(7,3)$, $(-1,-5)$, and $(3,-3)$?	
7	Find the trace of the matrix $\begin{bmatrix} 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \\ 4 & 8 & 12 & 16 \end{bmatrix}$.	
8	Find the projection of the vector $\begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$ onto the vector $\begin{bmatrix} -4 \\ -5 \\ 3 \end{bmatrix}$, with the vector written as a column vector and with entries written in decimal form.	
9	Find the value of the dot product $\begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \cdot \begin{bmatrix} -17 \\ 1.5 \\ 6 \end{bmatrix}$.	
10	When using Cramer's Rule on the system $\begin{cases} 2x+3y=117 \\ -2x+5y=40 \end{cases}$, the solution for x can be written as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find the product ab.	

6 points each		
11	Find the ordered pair solution to the system $\begin{cases} 14x-15y=10 \\ -15x+16y=-11 \end{cases}$.	
12	Find the element in the third row and second column for the inverse of the matrix $\begin{bmatrix} 2 & 3 & 5 \\ 4 & -6 & 1 \\ -1 & 4 & 1 \end{bmatrix}$.	
13	A square matrix A_n whose dimensions are $n \times n$ is defined by having each entry equal to the sum of its row and column numbers. For example, the element in the third row and second column of A_6 is $3+2=5$. Consider the determinants of A_2, A_3, A_4, ..., A_n, ... , where $n \geq 2$. Find the minimum value of all such determinants.	
14	Find the cross product, written as a column vector: $\begin{bmatrix} 2 \\ 5 \\ 11 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 13 \end{bmatrix}$	
15	What value of x makes the matrix $\begin{bmatrix} 4 & -2 & 1 \\ 3 & x & -6 \\ 2 & 5 & -3 \end{bmatrix}$ singular?	

2012 – 2013 Log1 Contest Round 1
Mu Matrices

Name: ________________

	4 points each	
1	Evaluate: $2\begin{bmatrix}1 & -3\\ 2 & 4\end{bmatrix}-3\begin{bmatrix}2 & -1\\ -1 & -4\end{bmatrix}$	
2	Evaluate: $\begin{vmatrix}-4 & 5\\ -5 & 6\end{vmatrix}$	
3	Solve for x: $\begin{bmatrix}x & 2 & 3\\ 7 & 6 & 8\end{bmatrix}\begin{bmatrix}5\\ 3\\ 1\end{bmatrix}=\begin{bmatrix}19\\ 61\end{bmatrix}$	
4	Consider matrix $A=\begin{bmatrix}-\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix}$. A rotates point (x,y) counterclockwise about the origin by some angle θ (in degrees) to point (a,b) by considering the equation $A\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}a\\ b\end{bmatrix}$. If θ is the least such positive such angle, find θ.	
5	A matrix A is called idempotent if $A^2=A$. If A is a 3×3 idempotent invertible matrix, find A.	

	5 points each	
6	How much area is enclosed by a triangle whose vertices are at the points $(7,3)$, $(-1,-5)$, and $(3,-3)$?	
7	Find the trace of the matrix $\begin{bmatrix}1 & 5 & 9 & 13\\ 2 & 6 & 10 & 14\\ 3 & 7 & 11 & 15\\ 4 & 8 & 12 & 16\end{bmatrix}$.	
8	Find the positive value of a that makes the vectors $\begin{bmatrix}2\\ 5\\ -1\\ -5\end{bmatrix}$ and $\begin{bmatrix}a^2\\ -a\\ 2\\ 1\end{bmatrix}$ perpendicular.	
9	Find the value of the dot product $\begin{bmatrix}2\\ -4\\ 7\end{bmatrix}\cdot\begin{bmatrix}-17\\ 1.5\\ 6\end{bmatrix}$.	
10	Find the absolute value of the sine of the angle between the vectors $\begin{bmatrix}4\\ -2\\ 3\end{bmatrix}$ and $\begin{bmatrix}-2\\ 1\\ 1\end{bmatrix}$.	

6 points each		
11	Find the ordered pair solution to the system $\begin{cases} 14x-15y=10 \\ -15x+16y=-11 \end{cases}$.	
12	Find the inverse of the matrix $\begin{bmatrix} 1 & 5 & 0 & 0 \\ 6 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$.	
13	A square matrix A_n whose dimensions are $n \times n$ is defined by having each entry equal to the sum of its row and column numbers. For example, the element in the third row and second column of A_6 is $3+2=5$. Consider the determinants of $A_2, A_3, A_4, \ldots, A_n, \ldots$, where $n \geq 2$. Find the minimum value of all such determinants.	
14	Find the cross product, written as a column vector: $\begin{bmatrix} 2 \\ 5 \\ 11 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 13 \end{bmatrix}$	
15	The equation of the plane containing the points $(2,-1,-1)$, $(3,4,-2)$, and $(0,1,-3)$ can be written as $Ax+By+Cz=D$, where A, B, C, and D are relatively prime integers with $A>0$. Find the value of $(A+D)^{B-C}$.	

2012 – 2013 Log1 Contest Round 1
Theta Matrices

Name: ________________

4 points each		
1	Evaluate: $2\begin{bmatrix}1 & -3\\2 & 4\end{bmatrix}-3\begin{bmatrix}2 & -1\\-1 & -4\end{bmatrix}$	$\begin{bmatrix}-4 & -3\\7 & 20\end{bmatrix}$
2	Evaluate: $\begin{vmatrix}-4 & 5\\-5 & 6\end{vmatrix}$	1
3	Solve for x: $\begin{bmatrix}x & 2 & 3\\7 & 6 & 8\end{bmatrix}\begin{bmatrix}5\\3\\1\end{bmatrix}=\begin{bmatrix}19\\61\end{bmatrix}$	2
4	Solve for x: $\begin{vmatrix}x & -6 & 2\\7 & 1 & 0\\1 & 3 & 5\end{vmatrix}=250$	0
5	If A is a 2×2 matrix whose inverse is equal to A, find A^{2012}.	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

5 points each		
6	How much area is enclosed by a triangle whose vertices are at the points $(7,3)$, $(-1,-5)$, and $(3,-3)$?	8
7	Find the trace of the matrix $\begin{bmatrix}1 & 5 & 9 & 13\\2 & 6 & 10 & 14\\3 & 7 & 11 & 15\\4 & 8 & 12 & 16\end{bmatrix}$.	34
8	Find the inverse of $\begin{bmatrix}-2 & 12\\0 & 4\end{bmatrix}$, with entries written as decimals.	$\begin{bmatrix}-0.5 & 1.5\\0 & 0.25\end{bmatrix}$
9	How many entries in the product $\begin{bmatrix}-3 & 7\\-2 & 0\\6 & -2\end{bmatrix}\begin{bmatrix}0 & 4 & 4 & -3 & 1\\1 & -2 & 0 & 2 & 1\end{bmatrix}$ are not equal to 0?	14
10	When using Cramer's Rule on the system $\begin{cases}2x+3y=117\\-2x+5y=40\end{cases}$, the solution for x can be written as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find the product ab.	7440

6 points each		
11	Find the ordered pair solution to the system $\begin{cases} 14x-15y=10 \\ -15x+16y=-11 \end{cases}$.	$(5,4)$
12	Find the element in the third row and second column for the inverse of the matrix $\begin{bmatrix} 2 & 3 & 5 \\ 4 & -6 & 1 \\ -1 & 4 & 1 \end{bmatrix}$.	$-\frac{11}{15}$
13	A square matrix A_n whose dimensions are $n \times n$ is defined by having each entry equal to the sum of its row and column numbers. For example, the element in the third row and second column of A_6 is $3+2=5$. Consider the determinants of A_2, A_3, A_4, ..., A_n, ..., where $n \geq 2$. Find the minimum value of all such determinants.	-1
14	If A, B, C, and D have dimensions of 2×6, 3×4, 2×3, and 6×4, respectively, which of the following is not a square matrix: $AD(CB)^T$, $(CB)^T AD$, $B^T B(AD)^T$, or $(A^T ADD^T)^T$?	$B^T B(AD)^T$
15	What value of x makes the matrix $\begin{bmatrix} 4 & -2 & 1 \\ 3 & x & -6 \\ 2 & 5 & -3 \end{bmatrix}$ singular?	$\frac{141}{14}$

2012 – 2013 Log1 Contest Round 1
Alpha Matrices

Name: ________________

	4 points each	
1	Evaluate: $2\begin{bmatrix}1 & -3\\ 2 & 4\end{bmatrix}-3\begin{bmatrix}2 & -1\\ -1 & -4\end{bmatrix}$	$\begin{bmatrix}-4 & -3\\ 7 & 20\end{bmatrix}$
2	Evaluate: $\begin{vmatrix}-4 & 5\\ -5 & 6\end{vmatrix}$	1
3	Solve for x : $\begin{bmatrix}x & 2 & 3\\ 7 & 6 & 8\end{bmatrix}\begin{bmatrix}5\\ 3\\ 1\end{bmatrix}=\begin{bmatrix}19\\ 61\end{bmatrix}$	2
4	Solve for x : $\begin{vmatrix}x & -6 & 2\\ 7 & 1 & 0\\ 1 & 3 & 5\end{vmatrix}=250$	0
5	A matrix A is called idempotent if $A^2=A$. If A is a 3×3 idempotent invertible matrix, find A.	$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

	5 points each	
6	How much area is enclosed by a triangle whose vertices are at the points $(7,3)$, $(-1,-5)$, and $(3,-3)$?	8
7	Find the trace of the matrix $\begin{bmatrix}1 & 5 & 9 & 13\\ 2 & 6 & 10 & 14\\ 3 & 7 & 11 & 15\\ 4 & 8 & 12 & 16\end{bmatrix}$.	34
8	Find the projection of the vector $\begin{bmatrix}2\\ -1\\ 5\end{bmatrix}$ onto the vector $\begin{bmatrix}-4\\ -5\\ 3\end{bmatrix}$, with the vector written as a column vector and with entries written in decimal form.	$\begin{bmatrix}-0.96\\ -1.2\\ 0.72\end{bmatrix}$
9	Find the value of the dot product $\begin{bmatrix}2\\ -4\\ 7\end{bmatrix}\cdot\begin{bmatrix}-17\\ 1.5\\ 6\end{bmatrix}$.	2
10	When using Cramer's Rule on the system $\begin{cases}2x+3y=117\\ -2x+5y=40\end{cases}$, the solution for x can be written as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find the product ab.	7440

6 points each		
11	Find the ordered pair solution to the system $\begin{cases} 14x-15y=10 \\ -15x+16y=-11 \end{cases}$.	$(5,4)$
12	Find the element in the third row and second column for the inverse of the matrix $\begin{bmatrix} 2 & 3 & 5 \\ 4 & -6 & 1 \\ -1 & 4 & 1 \end{bmatrix}$.	$-\frac{11}{15}$
13	A square matrix A_n whose dimensions are $n \times n$ is defined by having each entry equal to the sum of its row and column numbers. For example, the element in the third row and second column of A_6 is $3+2=5$. Consider the determinants of $A_2, A_3, A_4, \ldots, A_n, \ldots$, where $n \geq 2$. Find the minimum value of all such determinants.	-1
14	Find the cross product, written as a column vector: $\begin{bmatrix} 2 \\ 5 \\ 11 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 13 \end{bmatrix}$	$\begin{bmatrix} 32 \\ 51 \\ -29 \end{bmatrix}$
15	What value of x makes the matrix $\begin{bmatrix} 4 & -2 & 1 \\ 3 & x & -6 \\ 2 & 5 & -3 \end{bmatrix}$ singular?	$\frac{141}{14}$

2012 – 2013 Log1 Contest Round 1
Mu Matrices

Name: ________________

4 points each		
1	Evaluate: $2\begin{bmatrix}1 & -3\\ 2 & 4\end{bmatrix}-3\begin{bmatrix}2 & -1\\ -1 & -4\end{bmatrix}$	$\begin{bmatrix}-4 & -3\\ 7 & 20\end{bmatrix}$
2	Evaluate: $\begin{vmatrix}-4 & 5\\ -5 & 6\end{vmatrix}$	1
3	Solve for x: $\begin{bmatrix}x & 2 & 3\\ 7 & 6 & 8\end{bmatrix}\begin{bmatrix}5\\ 3\\ 1\end{bmatrix}=\begin{bmatrix}19\\ 61\end{bmatrix}$	2
4	Consider matrix $A=\begin{bmatrix}-\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix}$. A rotates point (x,y) counterclockwise about the origin by some angle θ (in degrees) to point (a,b) by considering the equation $A\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}a\\ b\end{bmatrix}$. If θ is the least such positive such angle, find θ.	210°
5	A matrix A is called idempotent if $A^2=A$. If A is a 3×3 idempotent invertible matrix, find A.	$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

5 points each		
6	How much area is enclosed by a triangle whose vertices are at the points $(7,3)$, $(-1,-5)$, and $(3,-3)$?	8
7	Find the trace of the matrix $\begin{bmatrix}1 & 5 & 9 & 13\\ 2 & 6 & 10 & 14\\ 3 & 7 & 11 & 15\\ 4 & 8 & 12 & 16\end{bmatrix}$.	34
8	Find the positive value of a that makes the vectors $\begin{bmatrix}2\\ 5\\ -1\\ -5\end{bmatrix}$ and $\begin{bmatrix}a^2\\ -a\\ 2\\ 1\end{bmatrix}$ perpendicular.	$\frac{7}{2}$
9	Find the value of the dot product $\begin{bmatrix}2\\ -4\\ 7\end{bmatrix}\cdot\begin{bmatrix}-17\\ 1.5\\ 6\end{bmatrix}$.	2
10	Find the absolute value of the sine of the angle between the vectors $\begin{bmatrix}4\\ -2\\ 3\end{bmatrix}$ and $\begin{bmatrix}-2\\ 1\\ 1\end{bmatrix}$.	$\frac{5\sqrt{870}}{174}$

6 points each		
11	Find the ordered pair solution to the system $\begin{cases} 14x-15y=10 \\ -15x+16y=-11 \end{cases}$.	$(5,4)$
12	Find the inverse of the matrix $\begin{bmatrix} 1 & 5 & 0 & 0 \\ 6 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$.	$\begin{bmatrix} 0 & 1/6 & 0 & 0 \\ 1/5 & -1/30 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/4 \end{bmatrix}$
13	A square matrix A_n whose dimensions are $n \times n$ is defined by having each entry equal to the sum of its row and column numbers. For example, the element in the third row and second column of A_6 is $3+2=5$. Consider the determinants of A_2, A_3, A_4, ..., A_n, ..., where $n \geq 2$. Find the minimum value of all such determinants.	-1
14	Find the cross product, written as a column vector: $\begin{bmatrix} 2 \\ 5 \\ 11 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 13 \end{bmatrix}$	$\begin{bmatrix} 32 \\ 51 \\ -29 \end{bmatrix}$
15	The equation of the plane containing the points $(2,-1,-1)$, $(3,4,-2)$, and $(0,1,-3)$ can be written as $Ax+By+Cz=D$, where A, B, C, and D are relatively prime integers with $A>0$. Find the value of $(A+D)^{B-C}$.	100

2012 – 2013 Log1 Contest Round 1
Matrices Solutions

Mu	Al	Th	Solution
1	1	1	$2\begin{bmatrix}1 & -3\\ 2 & 4\end{bmatrix}-3\begin{bmatrix}2 & -1\\ -1 & -4\end{bmatrix}=\begin{bmatrix}2 & -6\\ 4 & 8\end{bmatrix}-\begin{bmatrix}6 & -3\\ -3 & -12\end{bmatrix}=\begin{bmatrix}-4 & -3\\ 7 & 20\end{bmatrix}$
2	2	2	$\begin{vmatrix}-4 & 5\\ -5 & 6\end{vmatrix}=(-4)(6)-(-5)(5)=-24+25=1$
3	3	3	$\begin{bmatrix}19\\ 61\end{bmatrix}=\begin{bmatrix}x & 2 & 3\\ 7 & 6 & 8\end{bmatrix}\begin{bmatrix}5\\ 3\\ 1\end{bmatrix}=\begin{bmatrix}5x+6+3\\ 35+18+8\end{bmatrix}=\begin{bmatrix}5x+9\\ 61\end{bmatrix}\Rightarrow 19=5x+9\Rightarrow x=2$
	4	4	$250=\begin{vmatrix}x & -6 & 2\\ 7 & 1 & 0\\ 1 & 3 & 5\end{vmatrix}=5x+0+42-2-0-(-210)=5x+250\Rightarrow x=0$
4			The counterclockwise rotation matrix is $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, so $\cos\theta=-\frac{\sqrt{3}}{2}$ and $\sin\theta=-\frac{1}{2}$, and the smallest positive degree-measure angle satisfying these two equations is $\theta=210^\circ$.
		5	If $A=A^{-1}$, then $A^2=I\Rightarrow A^{2012}=\left(A^2\right)^{1006}=I^{1006}=I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
5	5		If A is idempotent and invertible, then $A=IA=\left(A^{-1}A\right)A=\left(A^{-1}\right)\left(A^2\right)=A^{-1}A=I$, so $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$.
6	6	6	The area is $\left\|\frac{1}{2}\begin{vmatrix}7 & 3 & 1\\ -1 & -5 & 1\\ 3 & -3 & 1\end{vmatrix}\right\|=\left\|\frac{1}{2}(-35+9+3-(-15)-(-21)-(-3))\right\|=\left\|\frac{1}{2}(16)\right\|=8$.
7	7	7	The trace is just the sum of the entries on the main diagonal: $1+6+11+16=34$.
		8	$\begin{bmatrix}-2 & 12\\ 0 & 4\end{bmatrix}^{-1}=\frac{1}{(-2)(4)-0(12)}\begin{bmatrix}4 & -12\\ 0 & -2\end{bmatrix}=\begin{bmatrix}-0.5 & 1.5\\ 0 & 0.25\end{bmatrix}$
	8		$\frac{\begin{bmatrix}2\\ -1\\ 5\end{bmatrix}\cdot\begin{bmatrix}-4\\ -5\\ 3\end{bmatrix}}{\begin{bmatrix}-4\\ -5\\ 3\end{bmatrix}\cdot\begin{bmatrix}-4\\ -5\\ 3\end{bmatrix}}\begin{bmatrix}-4\\ -5\\ 3\end{bmatrix}=\frac{-8+5+15}{16+25+9}\begin{bmatrix}-4\\ -5\\ 3\end{bmatrix}=\frac{12}{50}\begin{bmatrix}-4\\ -5\\ 3\end{bmatrix}=\begin{bmatrix}-0.96\\ -1.2\\ 0.72\end{bmatrix}$

8			$0=\begin{bmatrix}2\\5\\-1\\-5\end{bmatrix}\cdot\begin{bmatrix}a^2\\-a\\2\\1\end{bmatrix}=2a^2-5a-7=(2a-7)(a+1)\Rightarrow a=\frac{7}{2}$ since $a>0$
		9	The only entry that is 0 is the second row times the first column (it is easy to check that the other entries are not zeros). Since the dimensions of the result are 3×5, there are 14 nonzero entries.
9	9		$\begin{bmatrix}2\\-4\\7\end{bmatrix}\cdot\begin{bmatrix}-17\\1.5\\6\end{bmatrix}=(2)(-17)+(-4)(1.5)+(7)(6)=-34-6+42=2$
	10	10	$x=\dfrac{\begin{vmatrix}117 & 3\\40 & 5\end{vmatrix}}{\begin{vmatrix}2 & 3\\-2 & 5\end{vmatrix}}=\dfrac{585-120}{10-(-6)}=\dfrac{465}{16}$, which is in lowest terms, so $(465)(16)=7440$
10			$\sin\theta=\dfrac{\left\lVert\begin{bmatrix}4\\-2\\3\end{bmatrix}\times\begin{bmatrix}-2\\1\\1\end{bmatrix}\right\rVert}{\left\lVert\begin{bmatrix}4\\-2\\3\end{bmatrix}\right\rVert\cdot\left\lVert\begin{bmatrix}-2\\1\\1\end{bmatrix}\right\rVert}=\dfrac{\left\lVert\begin{bmatrix}-5\\-10\\0\end{bmatrix}\right\rVert}{\sqrt{16+4+9}\cdot\sqrt{4+1+1}}=\dfrac{\sqrt{25+100+0}}{\sqrt{29}\cdot\sqrt{6}}=\dfrac{5\sqrt{5}}{\sqrt{174}}=\dfrac{5\sqrt{870}}{174}$
11	11	11	$x=\dfrac{\begin{vmatrix}10 & -15\\-11 & 16\end{vmatrix}}{\begin{vmatrix}14 & -15\\-15 & 16\end{vmatrix}}=\dfrac{160-165}{224-225}=5$ and $y=\dfrac{\begin{vmatrix}14 & 10\\-15 & -11\end{vmatrix}}{\begin{vmatrix}14 & -15\\-15 & 16\end{vmatrix}}=\dfrac{-154+150}{224-225}=4$, so the ordered pair is $(5,4)$.
	12	12	$\dfrac{1}{\begin{vmatrix}2 & 3 & 5\\4 & -6 & 1\\-1 & 4 & 1\end{vmatrix}}\left(-\begin{vmatrix}2 & -1\\3 & 4\end{vmatrix}\right)=\dfrac{1}{-12-3+80-30-8-12}(-8-3)=-\dfrac{11}{15}$
12			$\begin{bmatrix}1 & 5 & 0 & 0\\6 & 0 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 4\end{bmatrix}^{-1}=\dfrac{1}{-240}\begin{bmatrix}0 & -40 & 0 & 0\\-48 & 8 & 0 & 0\\0 & 0 & -120 & 0\\0 & 0 & 0 & -60\end{bmatrix}=\begin{bmatrix}0 & 1/6 & 0 & 0\\1/5 & -1/30 & 0 & 0\\0 & 0 & 1/2 & 0\\0 & 0 & 0 & 1/4\end{bmatrix}$
13	13	13	For A_3 and above, two different choices of two consecutive rows can be subtracted to yield two rows that are identical, so all of those determinants are 0. $\lvert A_2\rvert=\begin{vmatrix}2 & 3\\3 & 4\end{vmatrix}=8-9=-1$, so the minimum value of all determinants is -1.

		14	$AD(CB)^T$ has dimensions $(2\times 6)(6\times 4)((2\times 3)(3\times 4))^T = (2\times 4)(2\times 4)^T = (2\times 4)(4\times 2)$ $= 2\times 2$, so this is square; likewise, $(CB)^T AD$ would have dimensions 4×4 and is thus square. $(A^T ADD^T)^T$ has dimensions $((6\times 2)(2\times 6)(6\times 4)(4\times 6))^T = (6\times 6)^T = 6\times 6$, which is also square. $B^T B(AD)^T$ has dimensions $(4\times 3)(3\times 4)((2\times 6)(6\times 4))^T$ $= (4\times 4)(2\times 4)^T = (4\times 4)(4\times 2) = 4\times 2$, so this is not square.
14	14		$\begin{bmatrix} 2 \\ 5 \\ 11 \end{bmatrix} \times \begin{bmatrix} 7 \\ 3 \\ 13 \end{bmatrix} = \begin{vmatrix} i & j & k \\ 2 & 5 & 11 \\ 7 & 3 & 13 \end{vmatrix} = 65i + 77j + 6k - 35k - 33i - 26j = 32i + 51j - 29k = \begin{bmatrix} 32 \\ 51 \\ -29 \end{bmatrix}$
	15	15	Singular is equivalent to meaning the determinant is 0, so $0 = \begin{vmatrix} 4 & -2 & 1 \\ 3 & x & -6 \\ 2 & 5 & -3 \end{vmatrix} = -12x + 24$ $+15 - 2x + 120 - 18 = -14x + 141 \Rightarrow x = \frac{141}{14}$.
15			Create two vectors in the plane by subtracting the points: $\langle 3-2, 4-(-1), -2-(-1) \rangle$ $= \langle 1,5,-1 \rangle$ and $\langle 0-2, 1-(-1), -3-(-1) \rangle = \langle -2,2,-2 \rangle$. The cross product of these vectors will be the normal vector to the plane and will thus give the coefficients of the variables. $\begin{vmatrix} i & j & k \\ 1 & 5 & -1 \\ -2 & 2 & -2 \end{vmatrix} = -10i + 2j + 2k + 10k + 2i + 2j = -8i + 4j + 12k$, so the equation of the plane is $-8x + 4y + 12z = D$. To find the value of D, plug in any point: $-8(0) + 4(1) + 12(-3) = 0 + 4 - 36 = -32$. Thus, the equation of the plane is $-8x + 4y + 12z = -32$, and dividing both sides by -4 yields $2x - y - 3z = 8$, and these coefficients are relatively prime integers with $A > 0$. Therefore, $(A+D)^{B-C}$ $= (2+8)^{-1-(-3)} = 10^2 = 100$.