2011 – 2012 Log1 Contest Round 2
Theta Number Theory

Name: ________________

4 points each		
1	How many prime numbers are less than 50?	
2	Twin primes are primes whose difference is 2. What is the largest two-digit twin prime pair?	
3	Find the product of the greatest common factor and the least common multiple of 14 and 30?	
4	Evaluate $11_2 + 111_3 + 1111_4$ and express as a base 5 number.	
5	Players of the game Mouse are awarded 0, 5 or 9 points on every turn. Some point totals, such as 11 are not possible. What is the largest point total that is not possible?	

5 points each		
6	How many zeroes are at the end of 111!?	
7	How many pairs of positive integers, m and n, are there such $7m + 5n = 400$?	
8	What is the probability that a randomly selected positive factor of 5,040 is even (express your answer as a reduced fraction)?	
9	What is the least common multiple of all the factors of 64?	
10	In 2011, the sum of the first two digits equals the sum of the last two digits. How many numbers between 2000 and 2500 satisfy this condition?	

6 points each		
11	Find the units digit of $3^{43}(5^{61}) + 7^{11}(11^4)$.	
12	Find the sum of the positive factors of 720.	
13	What is the hundreds digit of 2011^9?	
14	Given that $x \equiv 3 \bmod 7$, what is the remainder when $x^2 + 3x - 4$ is divided by 7?	
15	The digital root of the number is the sum of its digits taken until a sum less than 10 is obtained. For example, the digital root of 399 is 3 since 3+9+9=21 and then 2+1=3. What is the digital root of 10 factorial?	

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Alpha Number Theory

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4 points each		
1	How many prime numbers are less than 50?	
2	Twin primes are primes whose difference is 2. What is the largest two-digit twin prime pair?	
3	Find the product of the greatest common factor and the least common multiple of 14 and 30?	
4	Convert the base 3 number 211022_3 to base 9.	
5	Players of the game Mouse are awarded 0, 5 or 9 points on every turn. Some point totals, such as 11 are not possible. What is the largest point total that is not possible?	

5 points each		
6	How many zeroes are at the end of 111!?	
7	How many pairs of positive integers, *m* and *n*, are there such $7m + 5n = 400$?	
8	What is the probability that a randomly selected positive factor of 5,040 is even (express your answer as a reduced fraction)?	
9	A perfect number is a positive integer such that the sum of its proper positive factors equals itself. What is the sum of the second perfect number and the seventh prime number?	
10	In 2011, the sum of the first two digits equals the sum of the last two digits. How many numbers between 2000 and 2500 satisfy this condition?	

6 points each		
11	Find the units digit of $3^{43}(5^{61}) + 7^{11}(11^4)$.	
12	Find the sum of the positive factors of 720.	
13	What is the hundreds digit of 2011^9?	
14	How many positive integers less than or equal to 1000 are relatively prime (no common factors other than 1) to 100?	
15	The digital root of the number is the sum of its digits taken until a sum less than 10 is obtained. For example, the digital root of 399 is 3 since 3+9+9=21 and then 2+1=3. What is the digital root of 10 factorial?	

2011 – 2012 Log1 Contest Round 2
Mu Number Theory

Name: ________________

	4 points each	
1	How many prime numbers are less than 50?	
2	Twin primes are primes whose difference is 2. What is the largest two-digit twin prime pair?	
3	Find the product of the greatest common factor and the least common multiple of 14 and 30?	
4	Convert the base 3 number 211022_3 to base 9.	
5	Two digits, r and s, are such that the number 42r611s37 is divisible by 11. What is the remainder when r+s is divided by 11?	

	5 points each	
6	How many zeroes are at the end of 111!?	
7	How many pairs of positive integers, m and n, are there such $7m + 5n = 400$?	
8	What is the probability that a randomly selected positive factor of 5,040 is even (express your answer as a reduced fraction)?	
9	A perfect number is a positive integer such that the sum of its proper positive factors equals itself. What is the sum of the second perfect number and the seventh prime number?	
10	What is the smallest four-digit number that has exactly 3 positive factors?	

	6 points each	
11	Find the units digit of $3^{43}(5^{61}) + 7^{11}(11^4)$.	
12	Find the sum of the positive factors of 720.	
13	What is the hundreds digit of 2011^9?	
14	How many positive integers less than or equal to 1000 are relatively prime (no common factors other than 1) to 100?	
15	What is the sum of the integer values, n, so that $\frac{n^2+3n+3}{n-1}$ is also an integer?	

2011 – 2012 Log1 Contest Round 2
Theta Number Theory

Name: ________________

4 points each		
1	How many prime numbers are less than 50?	15
2	Twin primes are primes whose difference is 2. What is the largest two-digit twin prime pair?	71, 73
3	Find the product of the greatest common factor and the least common multiple of 14 and 30?	420
4	Evaluate $11_2 + 111_3 + 1111_4$ and express as a base 5 number.	$401_{[5]}$
5	Players of the game Mouse are awarded 0, 5 or 9 points on every turn. Some point totals, such as 11 are not possible. What is the largest point total that is not possible?	31

5 points each		
6	How many zeroes are at the end of 111!?	26
7	How many pairs of positive integers, m and n, are there such $7m + 5n = 400$?	11
8	What is the probability that a randomly selected positive factor of 5,040 is even (express your answer as a reduced fraction)?	$\frac{4}{5}$
9	What is the least common multiple of all the factors of 64?	64
10	In 2011, the sum of the first two digits equals the sum of the last two digits. How many numbers between 2000 and 2500 satisfy this condition?	25

6 points each		
11	Find the units digit of $3^{43}(5^{61}) + 7^{11}(11^4)$.	8
12	Find the sum of the positive factors of 720.	2418
13	What is the hundreds digit of 2011^9?	6
14	Given that $x \equiv 3 \bmod 7$, what is the remainder when $x^2 + 3x - 4$ is divided by 7?	0
15	The digital root of the number is the sum of its digits taken until a sum less than 10 is obtained. For example, the digital root of 399 is 3 since 3+9+9=21 and then 2+1=3. What is the digital root of 10 factorial?	9

2011 – 2012 Log1 Contest Round 2
Alpha Number Theory

Name: ________________

	4 points each	
1	How many prime numbers are less than 50?	15
2	Twin primes are primes whose difference is 2. What is the largest two-digit twin prime pair?	71, 73
3	Find the product of the greatest common factor and the least common multiple of 14 and 30?	420
4	Convert the base 3 number 211022_3 to base 9.	$738_{[9]}$
5	Players of the game Mouse are awarded 0, 5 or 9 points on every turn. Some point totals, such as 11 are not possible. What is the largest point total that is not possible?	31

	5 points each	
6	How many zeroes are at the end of 111!?	26
7	How many pairs of positive integers, m and n, are there such $7m + 5n = 400$?	11
8	What is the probability that a randomly selected positive factor of 5,040 is even (express your answer as a reduced fraction)?	$\frac{4}{5}$
9	A perfect number is a positive integer such that the sum of its proper positive factors equals itself. What is the sum of the second perfect number and the seventh prime number?	45
10	In 2011, the sum of the first two digits equals the sum of the last two digits. How many numbers between 2000 and 2500 satisfy this condition?	25

	6 points each	
11	Find the units digit of $3^{43}(5^{61}) + 7^{11}(11^4)$.	8
12	Find the sum of the positive factors of 720.	2418
13	What is the hundreds digit of 2011^9?	6
14	How many positive integers less than or equal to 1000 are relatively prime (no common factors other than 1) to 100?	400
15	The digital root of the number is the sum of its digits taken until a sum less than 10 is obtained. For example, the digital root of 399 is 3 since 3+9+9=21 and then 2+1=3. What is the digital root of 10 factorial?	9

2011 – 2012 Log1 Contest Round 2
Mu Number Theory

Name: ________________

4 points each		
1	How many prime numbers are less than 50?	15
2	Twin primes are primes whose difference is 2. What is the largest two-digit twin prime pair?	71, 73
3	Find the product of the greatest common factor and the least common multiple of 14 and 30?	420
4	Convert the base 3 number 211022_3 to base 9.	$738_{[9]}$
5	Two digits, r and s, are such that the number 42r611s37 is divisible by 11. What is the remainder when r+s is divided by 11?	0

5 points each		
6	How many zeroes are at the end of 111!?	26
7	How many pairs of positive integers, m and n, are there such $7m + 5n = 400$?	11
8	What is the probability that a randomly selected positive factor of 5,040 is even (express your answer as a reduced fraction)?	$\frac{4}{5}$
9	A perfect number is a positive integer such that the sum of its proper positive factors equals itself. What is the sum of the second perfect number and the seventh prime number?	45
10	What is the smallest four-digit number that has exactly 3 positive factors?	1369

6 points each		
11	Find the units digit of $3^{43}(5^{61}) + 7^{11}(11^4)$.	8
12	Find the sum of the positive factors of 720.	2418
13	What is the hundreds digit of 2011^9?	6
14	How many positive integers less than or equal to 1000 are relatively prime (no common factors other than 1) to 100?	400
15	What is the sum of the integer values, n, so that $\frac{n^2+3n+3}{n-1}$ is also an integer?	4

2011 – 2012 Log1 Contest Round 2
Number Theory Solutions

Mu	Al	Th	Solution
1	1	1	2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
2	2	2	Some may put 89,91 but 91=7*13.
3	3	3	The product of the GCF and the LCM will be the product of the two numbers. 14*30 = 420.
4	4		The base [9] is not needed. Since $9 = 3^2$, one can convert two base-3 digits directly into a base 9 digit.
		4	$3 + 13 + 85 = 101 = 4(25) + 1$
5			Divisibility for 11 means that (4+r+1+s+7)-(2+6+1+3) is a multiple of 11. This means that r+s is 0 or 11.
	5	5	5 x 9 – 5 – 9 = 31
6	6	6	111/5 = 22r1; 22/5=4r2; 4/5=0r4. 22+4+0=26
7	7	7	55(7)+3(5)=400, so (55,3) is one such pair. One can reduce m by 5 and increase n by 7 without changing the total. This continues until 5(7)+73(3). 11 total.
8	8	8	5040 is divisible by 16 or 2^4 so that 4/5 of the factors will be divisible by a power of 2 – we exclude 2^0=1. Thus 4/5 of them will be even. One can count the factors by completely factoring 5040 but it is not needed.
9	9		The first two perfect numbers are 6 and 28. 28 + 17=45
		9	The factors of 64 are all powers of 2 and divide into 64.
10			If a number has 3 factors, it is the square of a prime number. The first prime number with a 4-digit square is 37. $37^2 = 1369$.
	10	10	By exhaustion. Consider the first two digits 20 will go with 02, 20, and 11 (3 numbers). 21 will have 4 numbers, 22 (5 numbers) 23 (6 numbers) and 24 (7 numbers). 3+4+5+6+7 = 25.
11	11	11	The powers of 3 end respectively in 3, 9, 7, 1,... All powers of 5 end in 5, the powers of 7 in 7, 9, 3, 1 and finally all powers of 11 end in 1. Therefore the answer end in the same digit as 7(5)+3(1) = 38 with a units digit of 8.
12	12	12	$720 = 2^4 \times 3^2 \times 5$ The sum of the positive factors will be $(1 + 2^1 + 2^2 + 2^3 + 2^4)(1 + 3^1 + 3^2)(1 + 5^1) = (31)(13)(6) = 2418$
13	13	13	$2011^9 = (2000 + 11)^9 = 2000^9 + \cdots + 11^9$. All the terms except the last have at least 3 zeroes at the end. $11^9 = (10 + 1)^9 = 10^9 + \cdots + 9C7(10^2)(1^7) + 9C8(10)(1^8) + 1^9$. Only the last three terms need be considered (the rest have a factor 1000); 3600+900+1=3691 and the hundreds digit is 6.
14	14		The prime factors of 100 are 2 and 5. One needs to eliminate all multiples of 2 and 5. 1000(1/2)(4/5) = 400.
		14	$x^2 \equiv 9 \bmod 7$ and $3x \equiv 9 \bmod 7$ so $x^2 + 3x - 4 \equiv 9 + 9 - 4 \equiv 0 \bmod 7$,the remainder will be the 0.
15			$\frac{n^2+3n+3}{n-1} = n + 4 + \frac{7}{n-1}$ so that $\frac{7}{n-1}$ must also be an integer and $n - 1 = \pm 1, \pm 7$, $n = -6, 0, 2, or\ 8$. These sum to 4.
	15	15	The digital root is the remainder when divided by 9 (or 9 if it is divisible by 9). Since 10! is clearly divisible by 9, its digital root is 9.