2011 – 2012 Log1 Contest Round 3
Theta Individual

Name: _____

4 points each		
1	Find the median and mean for the following set of numbers: {8,10,16,3,8,6,12,4,20,13,11,14,5}	Median: ____ Mean: ____
2	Fred, in New York checked his thermometer and he saw that it was 45° Fahrenheit. He calls his friend, Sergei, in Russia and tells him that it was 45°. Sergei thought we meant 45° Celsius. In degrees Fahrenheit, what is 45° Celsius. There is linear relationship between Fahrenheit and Celsius: $0℃ = 32℉$ and $100℃ = 212℉$. Unit is optional.	
3	Find the sum of the number of faces, vertices, and edges of a pyramid whose base is in the shape of a regular pentagon	
4	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	
5	What is the sum $1 + 4 + 9 + 16 + \cdots + 100$?	

5 points each		
6	Solve for x: $512^{x+5} = 8^{x^2-3}$	
7	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	
8	Simplify the expression. $\frac{1}{2\times5} + \frac{1}{5\times8} + \frac{1}{8\times11} + \frac{1}{11\times14} + \frac{1}{14\times17}$	
9	If a@b means $\frac{\binom{a}{b}-\binom{3b}{a}}{a^b}$, evaluate 6@2, where $\binom{a}{b}$ means "a items choose b".	
10	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	

6 points each		
11	Find the coefficient of the x^6 term in the binomial expansion of $5x\,(3x-2)^7$.	
12	A bag contains 2 blue, 3 red and 5 green balls. Bill wants to know what the probability will be of picking a blue ball on his second draw. Previously drawn balls are not returned to the bag.	
13	9 points are arranged in a 3 row by 3 column pattern. What is the probability of randomly selecting three distinct points that form a triangle?	
14	$If\ A = \begin{bmatrix} 2 & -5 \\ 4 & -7 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -2 & 9 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -6 & -8 \end{bmatrix}$, find the value of A_{22}, the entry in the second row and second column of A.	
15	How many positive factors does the expression $20^2 11^2$ have?	

2011 – 2012 Log1 Contest Round 3
Alpha Individual

Name: _____

	4 points each	
1	Find the median and mean for the following set of numbers: {8,10,16,3,8,6,12,4,20,13,11,14,5}	Median: ____ Mean: ____
2	Fred, in New York checked his thermometer and he saw that it was 45° Fahrenheit. He calls his friend, Sergei, in Russia and tells him that it was 45°. Sergei thought we meant 45° Celsius. In degrees Fahrenheit, what is 45° Celsius. There is linear relationship between Fahrenheit and Celsius: $0℃ = 32℉$ and $100℃ = 212℉$. Unit is optional.	
3	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	
4	Solve for x: $512^{x+5} = 8^{x^2-3}$	
5	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	

	5 points each	
6	Simplify the expression. $\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\frac{1}{11\times14}+\frac{1}{14\times17}$	
7	The symbol $\binom{x}{y}$ means the number of ways one can choose y items from x distinguishable items. What is $\binom{2}{0}+\binom{3}{1}+\binom{4}{2}+\binom{5}{3}+\binom{6}{4}$?	
8	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	
9	A bag contains 2 blue, 3 red and 5 green balls. Bill wants to know what the probability will be of picking a blue ball on his second draw. Previously drawn balls are not returned to the bag.	
10	Dave and Molly are playing a game where they take turns spinning a spinner. The spinner has a one-third chance of coming up "WIN" and two-thirds chance of "PASS". Dave goes first and they take turns until one of them spins "WIN". What is the probability that Dave wins?	

	6 points each	
11	$If\ A = \begin{bmatrix}2 & -5\\ 4 & -7\end{bmatrix} + \begin{bmatrix}3 & -1\\ -2 & 9\end{bmatrix}\begin{bmatrix}0 & 4\\ -6 & -8\end{bmatrix}$, find the value of A_{22}, the entry in the second row and second column of A.	
12	How many positive factors does the expression $20^2 11^2$ have?	
13	Find the number of common prime factors of 2002 and 1729.	
14	Convert this equation from polar to rectangular form. $r = 4\sin\theta$	
15	Evaluate $\cos(75°) \times \sqrt{\sin(30°)}$	

2011 – 2012 Log1 Contest Round 3
Mu Individual

Name: _____

4 points each		
1	Fred, in New York checked his thermometer and he saw that it was 45° Fahrenheit. He calls his friend, Sergei, in Russia and tells him that it was 45°. Sergei thought we meant 45° Celsius. In degrees Fahrenheit, what is 45° Celsius. There is linear relationship between Fahrenheit and Celsius: 0°C = 32°F and 100°C = 212°F. Unit is optional.	
2	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	
3	Solve for x: $512^{x+5} = 8^{x^2-3}$	
4	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	
5	Simplify the expression. $\frac{1}{2\times5} + \frac{1}{5\times8} + \frac{1}{8\times11} + \frac{1}{11\times14} + \frac{1}{14\times17}$	

5 points each		
6	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	
7	Find the coefficient of the x^6 term in the binomial expansion of $5x\,(3x-2)^7$.	
8	9 points are arranged in a 3 row by 3 column pattern. What is the probability of randomly selecting three distinct points that form a triangle?	
9	There are 5 coins in a box. 2 have a 40% chance of landing on heads and the other 3 are fair coins. If you pick two at random and flip them, what is the probability that they will both be tails? Express as a percentage.	
10	How many positive factors does the expression $20^2 11^2$ have?	

6 points each		
11	What is the smallest positive three-digit number that has a remainder of 2 when divided by 3, a remainder of 4 when divided by 5 and a remainder of 6 when divided by 7?	
12	Convert this equation from polar to rectangular form. $r = 4\sin\theta$	
13	Evaluate $\cos(75°) \times \sqrt{\sin(30°)}$	
14	Given the function $7xe^{2y} = 2$, determine the value of $\frac{dy}{dx}$ when $x = 5$.	
15	Calculate the volume of an object that is produced when the function $f(x) = x^2 + 3x + 2$ on the domain $x = [0, 4]$ is revolved around the x-axis.	

2011 – 2012 Log1 Contest Round 3
Theta Individual

Name: _____

	4 points each	
1	Find the median and mean for the following set of numbers: {8,10,16,3,8,6,12,4,20,13,11,14,5}	Median: 10 Mean: 10
2	Fred, in New York checked his thermometer and he saw that it was 45° Fahrenheit. He calls his friend, Sergei, in Russia and tells him that it was 45°. Sergei thought we meant 45° Celsius. In degrees Fahrenheit, what is 45° Celsius. There is linear relationship between Fahrenheit and Celsius: $0°C = 32°F$ and $100°C = 212°F$. Unit is optional.	113
3	Find the sum of the number of faces, vertices, and edges of a pyramid whose base is in the shape of a regular pentagon	22
4	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	1
5	What is the sum $1 + 4 + 9 + 16 + \cdots + 100$?	385

	5 points each	
6	Solve for x: $512^{x+5} = 8^{x^2-3}$	{6,-3}
7	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	4:42
8	Simplify the expression. $\frac{1}{2\times5} + \frac{1}{5\times8} + \frac{1}{8\times11} + \frac{1}{11\times14} + \frac{1}{14\times17}$	$\frac{5}{34}$
9	If a@b means $\frac{\binom{a}{b}-\binom{3b}{a}}{a^b}$, evaluate 6@2.	$\frac{7}{18}$
10	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	900,900

	6 points each	
11	Find the coefficient of the x^6 term in the binomial expansion of $5x\,(3x-2)^7$.	102060
12	A bag contains 2 blue, 3 red and 5 green balls. Bill wants to know what the probability will be of picking a blue ball on his second draw. Previously drawn balls are not returned to the bag.	$\frac{1}{5}$
13	9 points are arranged in a 3 row by 3 column pattern. What is the probability of randomly selecting three distinct points that form a triangle?	$\frac{19}{21}$
14	$If\ A = \begin{bmatrix} 2 & -5 \\ 4 & -7 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -2 & 9 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -6 & -8 \end{bmatrix}$, find the value of A_{22}, the entry in the second row and second column of A.	$A_{22} = -87$
15	How many positive factors does the expression $20^2 11^2$ have?	45

2011 – 2012 Log1 Contest Round 3
Alpha Individual

Name: _____

	4 points each	
1	Find the median and mean for the following set of numbers: {8,10,16,3,8,6,12,4,20,13,11,14,5}	Median: 10 Mean: 10
2	Fred, in New York checked his thermometer and he saw that it was 45° Fahrenheit. He calls his friend, Sergei, in Russia and tells him that it was 45°. Sergei thought we meant 45° Celsius. In degrees Fahrenheit, what is 45° Celsius. There is linear relationship between Fahrenheit and Celsius: $0°C = 32°F$ and $100°C = 212°F$. Unit is optional.	113
3	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	1
4	Solve for x: $512^{x+5} = 8^{x^2-3}$	{6,-3}
5	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	4:42

	5 points each	
6	Simplify the expression. $\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\frac{1}{11\times14}+\frac{1}{14\times17}$	$\frac{5}{34}$
7	The symbol $\binom{x}{y}$ means the number of ways one can choose y items from x distinguishable items. What is $\binom{2}{0}+\binom{3}{1}+\binom{4}{2}+\binom{5}{3}+\binom{6}{4}$?	35
8	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	900,900
9	A bag contains 2 blue, 3 red and 5 green balls. Bill wants to know what the probability will be of picking a blue ball on his second draw. Previously drawn balls are not returned to the bag.	$\frac{1}{5}$
10	Dave and Molly are playing a game where they take turns spinning a spinner. The spinner has a one-third chance of coming up "WIN" and two-thirds chance of "PASS". Dave goes first and they take turns until one of them spins "WIN". What is the probability that Dave wins?	$\frac{3}{5}$

	6 points each	
11	$If\ A = \begin{bmatrix}2 & -5\\ 4 & -7\end{bmatrix} + \begin{bmatrix}3 & -1\\ -2 & 9\end{bmatrix}\begin{bmatrix}0 & 4\\ -6 & -8\end{bmatrix}$, find the value of A_{22}, the entry in the second row and second column of A.	A_{22} = -87
12	How many positive factors does the expression $20^2 11^2$ have?	45
13	Find the number of common prime factors of 2002 and 1729.	2
14	Convert this equation from polar to rectangular form. $r = 4\sin\theta$	$x^2 + y^2 = 4y$ or equivalent
15	Evaluate $\cos(75°) \times \sqrt{\sin(30°)}$	$\frac{\sqrt{3}-1}{4}$ or equivalent

2011 – 2012 Log1 Contest Round 3
Mu Individual

Name: _____

	4 points each	
1	Fred, in New York checked his thermometer and he saw that it was 45° Fahrenheit. He calls his friend, Sergei, in Russia and tells him that it was 45°. Sergei thought we meant 45° Celsius. In degrees Fahrenheit, what is 45° Celsius. There is linear relationship between Fahrenheit and Celsius: 0°C = 32°F and 100°C = 212°F. Unit is optional.	113
2	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	1
3	Solve for x: $512^{x+5} = 8^{x^2-3}$	{6,-3}
4	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	4:42
5	Simplify the expression. $\frac{1}{2\times5} + \frac{1}{5\times8} + \frac{1}{8\times11} + \frac{1}{11\times14} + \frac{1}{14\times17}$	$\frac{5}{34}$

	5 points each	
6	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	900,900
7	Find the coefficient of the x^6 term in the binomial expansion of $5x\,(3x-2)^7$.	102060
8	9 points are arranged in a 3 row by 3 column pattern. What is the probability of randomly selecting three distinct points that form a triangle?	19/21
9	There are 5 coins in a box. 2 have a 40% chance of landing on heads and the other 3 are fair coins. If you pick two at random and flip them, what is the probability that they will both be tails? Express as a percentage.	29.1%
10	How many positive factors does the expression $20^2 11^2$ have?	45

	6 points each	
11	What is the smallest positive three-digit number that has a remainder of 2 when divided by 3, a remainder of 4 when divided by 5 and a remainder of 6 when divided by 7?	104
12	Convert this equation from polar to rectangular form. $r = 4\sin\theta$	$x^2 + y^2 = 4y$ or equivalent
13	Evaluate $\cos(75°) \times \sqrt{\sin(30°)}$	$\frac{\sqrt{3}-1}{4}$ or equivalent
14	Given the function $7xe^{2y} = 2$, determine the value of $\frac{dy}{dx}$ when $x = 5$.	$-\frac{1}{10}$
15	Calculate the volume of an object that is produced when the function $f(x) = x^2 + 3x + 2$ on the domain x = [0, 4] is revolved around the x-axis.	$\frac{14672\pi}{15}$

2011 – 2012 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
	1	1	Median: 10 Mean: $130/13 = 10$ {3,4,5,6,8,8,10,11,12,13,14,16,20}
1	2	2	Convert Celsius to Fahrenheit $F = \frac{9}{5}C + 32 \;\; F = \frac{9}{5}(45) + 32 \;\; F = \frac{9}{5}(45) + 32 \;\; F = 113$
		3	This pyramid has 6 vertices. Each edge on the base pentagon corresponds to exactly one lateral face on the pyramid. Thus there are 6 faces on this pyramid (including the base). Each vertex on the base pentagon corresponds to exactly one lateral edge on the pyramid. Thus there are 5 lateral edges on the pyramid and 5 edges on the base pentagon. The sum is 22.
2	3	4	The difference between the 11th and 7th elements is 4 times the common difference. The 7th and 1st elements differ by 6 times the common difference. The 1st element is then 22 – (6/4)(36-22) = 22-21 = 1
		5	The formula is n(n+1)(2n+1)/6 = (10)(11)21/6 = 385.
3	4	6	$512^{x+5} = 8^{x^2-3} = (2^9)^{x+5} = (2^3)^{x^2-3}$ $9(x+5) = 3(x^2-3)$ $3(x+5) = (x^2-3)$ $3x + 15 = x^2 - 3$ $0 = x^2 - 3x - 18$ $0 = (x-6)(x+3)$ $x = \{6, -3\}$
4	5	7	There are 3*365*24 = 26280 hours in exactly three year. Thus, 26280 seconds are lost. This is equivalent to 438 minutes or 7 hours, 18 minutes. Exactly three years later, instead of reading midnight, the clock will be slow by this amount, equivalent to 4:42.
5	6	8	Knowing that $\frac{C}{AB} = \left(\frac{C}{B-A}\right) \times \left(\frac{1}{A} - \frac{1}{B}\right)$ $\left(\frac{1}{3}\right) \times \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{3}\right) \times \left(\frac{1}{5} - \frac{1}{8}\right) + \cdots + \left(\frac{1}{3}\right) \times \left(\frac{1}{14} - \frac{1}{17}\right)$ Factoring out the common 1/3, only the first and last terms remain. All the middle terms cancel out. Therefore, $\left(\frac{1}{3}\right) \times \left(\frac{1}{2} - \frac{1}{17}\right) = \frac{1}{3}\left(\frac{15}{34}\right) = \frac{5}{34}$
		9	Evaluate. $\frac{\binom{6}{2} - \binom{3x2}{6}}{6^2} = \frac{15-1}{36} = \frac{14}{36} = \frac{7}{18}$
	7		One can evaluate each combination separately but there is a formula sometimes referred to as "hockey-stick" which has this sum equal $\binom{7}{4} = 35$. In other words, $\binom{n}{r} = \sum_{j=o}^{r} \binom{n+j-r-1}{j}$
6	8	10	13! / (4! 4! 3! 2!) = 900,900

7		11	For the factor $(3x-2)^7$, the coefficient of the x^5 term is $3^5(-2)^2\binom{7}{5} = 243(4)(21) = 20412$. This number is then multiplied by 5 to attain the final result of 102060
	9	12	$\lfloor P(R1 or G1)\ AND\ P(B2) \rfloor\ or\ \lfloor P(B1) AND\ P(B2) \rfloor$ $\left\lfloor \frac{8}{10} \times \frac{2}{9} \right\rfloor + \left\lfloor \frac{2}{10} \times \frac{1}{9} \right\rfloor = \frac{18}{90} = \frac{1}{5}$
8		13	There are $\binom{9}{3} = 84$ ways to choose 3 points out of a 3x3 lattice. Of these 84 points, the only possible combinations that CANNOT form a triangle are those that are collinear. These would be the 3 columns, 3 rows and 2 diagonal sets of points. Thus there are 76 possible combinations that can form a triangle. The probability is $\frac{76}{84} = \frac{19}{21}$
	10		In the first turn, Dave has a 1/3 chance of winning. In order for Molly to win, Dave must pass and then Molly "win" with probability $(2/3)(1/3) = 2/9$. There is a 5/9 chance of someone winning in the first round and 4/9 chance that it goes to round two. One can treat this as an infinite sequence or note that Dave's probability of winning is $\frac{1}{3} \div \frac{5}{9} = \frac{3}{5}$, so the desired chance is 3/5.
9			Create a choice table: H 50 50 50 40 40 T 50 50 50 60 60 Define the following probabilities: P(TT) = probability of throwing two tails regardless of the type of coin. $P(T_{50}T_{50})$ = probability of drawing 2 regular coins AND throwing two tails. $\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{40}$ $P(T_{60}T_{60})$ = probability of drawing 2 biased coins AND throwing two tails. $\left(\frac{2}{5}\right)\left(\frac{1}{4}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) = \frac{9}{250}$ $P(T_{50}T_{60})$ = probability of drawing a biased and a regular coin AND throwing two tails. $\left(\frac{6}{10}\right)\left(\frac{1}{2}\right)\left(\frac{3}{5}\right) = \frac{9}{50}$ There are 6 out of 10 possible ways to draw a biased and a regular coin. $P(TT) = P(T_{50}T_{50}) + P(T_{60}T_{60}) + P(T_{50}T_{60})$ $P(TT) = \frac{3}{40} + \frac{9}{250} + \frac{9}{50} = \frac{75+36+180}{1000} = \frac{291}{1000}$
	11	*14*	$Let\ C = \begin{bmatrix} 3 & -1 \\ -2 & 9 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -6 & -8 \end{bmatrix}$ $Then\ C = \begin{bmatrix} 3(0)-1(-6) & 3(4)-1(-8) \\ -2(0)+9(-6) & -2(4)+9(-8) \end{bmatrix}$ $Thus\ C = \begin{bmatrix} 6 & 20 \\ -54 & -80 \end{bmatrix}$ Evaluating $A = \begin{bmatrix} 2 & -5 \\ 4 & -7 \end{bmatrix} + \begin{bmatrix} 6 & 20 \\ -54 & -80 \end{bmatrix} = \begin{bmatrix} 8 & 15 \\ -50 & -87 \end{bmatrix}$ For this problem, one really only needs: $-7 + (-2)(4) + 9(-8) = -87$
10	12	15	In terms of prime factors $20^2 11^2 = 2^4 5^2 11^2$. Therefore, the total number of factors is $(4+1)(2+1)(2+1) = 45$
	13		One can find the greatest common divisor using the Euler method. 2002=1729(1) + 273, 1729=273(6)+91 and 273=91(3). This means the GCF=91 with prime factors of 7 and 13.
11			If 1 is added to the number, it is divisible by 3, 5 and 7 or 3*5*7=105. Therefore the number is 105-1 = 104.

12	14		Since $r^2 = x^2 + y^2$ and $y = r\ \sin\theta$. The equation $r = 4\sin\theta$, becomes $r^2 = 4\,r\sin\theta = 4y$. Thus, $x^2 + y^2 = 4y$ Other possibilities $x^2 + (y-2)^2 = 4$ or $x^2 + y^2 - 4y = 0$. It is a circle of radius 2 centered at (0,2).
13	15		The answer will depend on whether a sum formula or half angle formula is used to evaluate cos(75). I. $\cos(75) = \cos(45+30) = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$, so the answer is $\frac{\sqrt{6}-\sqrt{2}}{4}\frac{\sqrt{2}}{2} = \frac{\sqrt{3}-1}{4}$ II. Let $A = \cos(75)\sqrt{\sin(30)}$ $A = \sin(15)\sqrt{\sin(30)} = \sqrt{\frac{1-\cos(30)}{2}}\sqrt{\frac{1}{2}}$ Therefore, $A = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{4}} = \sqrt{\frac{2-\sqrt{3}}{8}} = \frac{1}{2}\sqrt{\frac{2-\sqrt{3}}{2}} = \frac{1}{2}\sqrt{1-\frac{\sqrt{3}}{2}}$
14			Implicitly differentiating, $\frac{d}{dx}(7xe^{2y}) = \frac{d}{dx}(2)$ $7xe^{2y}(2)\frac{dy}{dx} + e^{2y}(7) = 0$ $14xe^{2y}\frac{dy}{dx} = -7e^{2y}$ $14x\frac{dy}{dx} = -7$ $\frac{dy}{dx} = -\frac{7}{14x} = -\frac{7}{14(5)} = -\frac{1}{10}$
15			Using the disc method, $V = \int_0^4 \pi f(x)^2 dx$ $V = \int_0^4 \pi\,(x^2+3x+2)^2 dx$ $V = \pi\int_0^4 (x^4+6x^3+13x^2+12x+4)\,dx$ $V = \pi\left[\frac{1}{5}x^5 - \frac{3}{2}x^4 + \frac{13}{3}x^3 + 6x^2 + 4x \;\Big|_0^4\right]$ $V = \pi\left[\frac{1024}{5} + \frac{768}{2} + \frac{832}{3} + \frac{192}{2} + \frac{32}{2}\right]$ $V = \pi\left[\frac{6144+11520+8320+2880+480}{30}\right]$ $V = \pi\left[\frac{29344}{30}\right]$ $V = \pi\left[\frac{14672}{15}\right]$