2011 – 2012 Log1 Contest Round 1
Theta Equations / Inequalities

Name: _____
Units are not required

	4 points each	
1	Solve for x: $6(x-3)+4x+6=3(2x+1)+5$	
2	For how many integers, *n*, is the following inequality true? $\lvert n-10\rvert \leq 12$	
3	Berta, a taxi driver, charges a \$2.25 flat rate in addition to \$0.75 per mile. Stacie wants to attend a festival 32 miles but has no more than \$15.50 to spend on a ride. Assuming that the number of miles Berta drives must be an integer, how many miles will Stacie have to walk to the festival?	
4	On what interval is $x^2-x\leq 6$?	
5	For how many integer values, *k*, does the equation $x^2-kx+k=0$ have non-real solutions?	

	5 points each	
6	When $(x+2y)^4$ is expanded, what is the sum of the coefficients?	
7	What is the maximum value of *y*, if $y=-2x^2+12x+13$?	
8	What values of x satisfy $\sqrt{3x+\sqrt{3x+\sqrt{3x+\cdots}}}=6$?	
9	If $f(x)=\frac{x+1}{x-3}$, what is $f^{-1}(5)$?	
10	How many real solutions does the equation $x^4+6x^2-16=0$ have?	

	6 points each	
11	Suppose $f(x)=x^3+4x^2+10x+7$ and $g(x)=x^2+3x+7$. Simplify $\frac{f(x)}{g(x)}$.	
12	If $a+b+c=6, 2a-b+3c=8, 4a-2b+c=-9$, then what is $a-b-c$?	
13	What is the maximum value of $5x-2y$ if (x,y) is in the region bounded by $x\geq 0, y\geq 0$ and $3x+4y\leq 12$?	
14	Solve for x: $4^{x+3}8^{2x}=16^{3x-1}$	
15	What is the sum of the squares of the roots of the equation$2x^2-3x+5=0$?	

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14	The polar equation of a spiral $r=2^{\frac{\theta}{2\pi}}$ for $\theta\epsilon[0,15\pi]$ intersects the segment, in (r,θ) form, of (10,0) to (1000,0) in how many points?	
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Mu Equations / Inequalities

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5	For how many integer values, *k*, does the equation $x^3-(k+1)x^2+2kx-k=0$ have non-real solutions?	

	5 points each	
6	When $(x+2y)^4$ is expanded, what is the sum of the coefficients?	
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9	If $f(x)=\log\left(log\frac{x+6}{x-1}\right)$, what is $f^{-1}(0)$	
10	What is the maximum value of $y=x^3-3x+4$ on the interval $x\epsilon[-2,3]$?	

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11	Suppose $f(x)=x^3+4x^2+10x+7$ and $g(x)=x^2+3x+7$. Simplify $\frac{f(x)}{g(x)}$.	
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15	If $\ln(x+y)=\sin(xy)$, what is $\frac{dy}{dx}$ when $x=0, y=1$?	

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1	Solve for x: $6(x-3)+4x+6=3(2x+1)+5$	5
2	For how many integers, *n*, is the following inequality true? $\lvert n-10\rvert \leq 12$	25
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4	On what interval is $x^2-x\leq 6$?	$-2\leq x\leq 3$ or $[-2,3]$
5	For how many integer values, *k*, does the equation $x^2-kx+k=0$ have non-real solutions?	3

	5 points each	
6	When $(x+2y)^4$ is expanded, what is the sum of the coefficients?	81
7	What is the maximum value of *y*, if $y=-2x^2+12x+13$?	31
8	What values of x satisfy $\sqrt{3x+\sqrt{3x+\sqrt{3x+\cdots}}}=6$?	[x=] 10
9	If $f(x)=\frac{x+1}{x-3}$, what is $f^{-1}(5)$?	4
10	How many real solutions does the equation $x^4+6x^2-16=0$ have?	2

	6 points each	
11	Suppose $f(x)=x^3+4x^2+10x+7$ and $g(x)=x^2+3x+7$. Simplify $\frac{f(x)}{g(x)}$.	$\left[\frac{f(x)}{g(x)}\right]$ $=x+1$
12	If $a+b+c=6, 2a-b+3c=8, 4a-2b+c=-9$, then what is $a-b-c$?	-10
13	What is the maximum value of $5x-2y$ if (x,y) is in the region bounded by $x\geq 0, y\geq 0$ and $3x+4y\leq 12$?	20
14	Solve for x: $4^{x+3}8^{2x}=16^{3x-1}$	$[x=]\ \frac{5}{2}$
15	What is the sum of the squares of the roots of the equation $2x^2-3x+5=0$?	$-\frac{11}{4}$

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9	If $f(x)=\log\left(log\frac{x+6}{x-1}\right)$, what is $f^{-1}(0)$	$\frac{16}{9}$
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11	Suppose $f(x)=x^3+4x^2+10x+7$ and $g(x)=x^2+3x+7$. Simplify $\frac{f(x)}{g(x)}$.	$\left[\frac{f(x)}{g(x)}\right]=x+1$
12	If $a+b+c=6, 2a-b+3c=8, 4a-2b+c=-9$, then what is $a-b-c$?	-10
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	5 points each	
6	When $(x+2y)^4$ is expanded, what is the sum of the coefficients?	81
7	What is the maximum value of *y*, if $y=-2x^2+12x+13$?	31
8	What values of x satisfy $\sqrt{3x+\sqrt{3x+\sqrt{3x+\cdots}}}=6$?	[x=] 10
9	If $f(x)=\log\left(log\frac{x+6}{x-1}\right)$, what is $f^{-1}(0)$	$\frac{16}{9}$
10	What is the maximum value of $y=x^3-3x+4$ on the interval $x\epsilon[-2,3]$?	22

	6 points each	
11	Suppose $f(x)=x^3+4x^2+10x+7$ and $g(x)=x^2+3x+7$. Simplify $\frac{f(x)}{g(x)}$.	$\left[\frac{f(x)}{g(x)}\right]$ $=x+1$
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15	If $\ln(x+y)=\sin(xy)$, what is $\frac{dy}{dx}$ when $x=0, y=1$?	0

2011 – 2012 Log1 Contest Round 1
Equations / Inequalities Solutions

Mu	Al	Th	Solution
1	1	1	$6x-18+4x+6=6x+3+5$ $10x-12=6x+8$ $4x=20$ $x=5$
2	2	2	The absolute value of n minus 10 is the distance from n to 10. There are 12 integers greater and 12 less and the number 10 itself for a total of 25 integers.
3	3	3	Subtracting the fixed cost of 2.25, Stacie has 13.25 to spend on miles. That will take her 17+ miles, leaving her 15 (of the 32) miles to walk.
4	4		The equation factors into $\sin(x)\left(\frac{1}{2}+\cos(x)\right)=0$, which has solutions when $x=0,\pi,\frac{2\pi}{3},\frac{4\pi}{3}$, for a total of 4 solutions.
		4	$x^2-x-6\leq 0,(x+2)(x-3)\leq 0$. This is a parabola open upwards; it will be negative between the zeros. We need to include the zeros as the inequality includes 6.
5			First, notice that x=1 is a solution and factor it out. The polynomial left is the equation for the Alphas and Thetas.
	5	5	The discriminant is k^2-4k and is negative only for k=1, 2, and 3.
6	6	6	If x=y=1, then the expanded polynomial will equal the sum of the coefficients. $\left(1+2(1)\right)^4=3^4=81$
7	7	7	The maximum will occur at $x=-\frac{b}{2a}=3$. Its value will be $-2(9)+12(3)+13=31$
8	8	8	$6=\sqrt{3x+\sqrt{3x+\sqrt{3x+\cdots}}}=\sqrt{3x+6}$ $36=3x+6,x=10$
9	9		Equivalent to solving: $10^{10^0}=\frac{x+6}{x-1},10x-10=x+6,x=\frac{16}{9}$
		9	Equivalent to solving: $5=\frac{x+1}{x-3},5x-15=x+1,x=4.$
10			Taking the derivative: $3x^2-3=0,x=-1,1.$ One must test these points and the endpoints -2 and 3 to get the maximum value at x=3 of 22.
	10	10	The equation has solutions of $x^2=-8\ and\ x^2=2$. Only the latter equation has real solutions.
11	11	11	Use long division. $\frac{x^3+4x^2+10x+7}{x^2+3x+7}=\frac{x(x^2+3x+7)+(x^2+3x+7)}{x^2+3x+7}=x+1$
12	12	12	Adding the 1st and 2nd equations, and twice the first to the 3rd equation, we get: $3a+4c=14,6a+3c=3$. Of these subtract the 2nd from twice the 1st and get: $5c=25,c=5$. Back substitute to get: $a=-2,b=3;a-b-c=-10$.
13	13	13	The maximum and minimum will occur at one of the corners of the region, (0,0), (4,0) and (0,3). The maximum of 20 occurs at (4,0).
14	14		The distance r, will have to be a power of 2 between 10 and 1000, namely 16, 32, 64, 128. 128 occurs at 14π. A total of 4 points.

		14	Change to a base of 2 and equate exponents. $(2^2)^{x+3}(2^3)^{2x} = (2^4)^{3x-1}$ $2^{2x+6}2^{6x} = 2^{12x-4}$ $8x + 6 = 12x - 4, x = \frac{5}{2}$
15			Taking derivative implicitly, $\frac{1}{x+y}\left(1 + \frac{dy}{dx}\right) = \cos(xy)\left(y + \frac{dy}{dx}x\right)$ $\frac{1}{0+1}\left(1 + \frac{dy}{dx}\right) = \cos(0 \cdot 1)\,(1 + 0)$ $\frac{dy}{dx} = 0$
	15	15	If the two roots are r and s, then $(r+s)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4} = r^2 + s^2 + 2rs$ $\frac{9}{4} = r^2 + s^2 + 2\left(\frac{5}{2}\right)$ $r^2 + s^2 = \frac{9}{4} - 5 = -\frac{11}{4}$