

**2010 – 2011 Log1 Contest Round 2**  
**Theta Equations and Inequalities**

Name: \_\_\_\_\_

4 points each	
1	Solve for x: $5(x - 2) - (x - 4) = 3(x - 1) - 2(x + 1)$
2	Bertha has a strange farm that only has zebras and ostriches. When she looks out to the field she counts 35 heads and 102 legs. How many zebras does she have?
3	What is the sum of the roots of $x(2 - 3x)(4x^2 - 3x + 5) = 0$ ?
4	One solution to the equation $x^2 - 3x + 1 = 0$ is $\frac{3 - \sqrt{5}}{2}$ , what is the other solution?
5	Find $(f \circ g)(-3)$ if $f(x) = \sqrt{x + 16}$ and $g(x) = 3x^2 - 2x + 15$ .

5 points each	
6	How many integers are solutions to $ 2x + 1  \leq 5$ ?
7	If $y$ is inversely proportional to the square of $x$ , and $y = 1/2$ when $x = 10$ , then what is $x$ , greater than 0, when $y = 1/8$ ?
8	Find the inverse of $y = \sqrt{2 - 7x}$ , when $x \leq \frac{2}{7}$ .
9	If $9^x = 7$ and $7^y = 3$ , what is $xy$ ?
10	What is the equation of a line perpendicular to $3x - 2y = 12$ that goes through the point $(-2, 4)$ ? State answer in the form of $Ax + By = C$ .

6 points each	
11	Find the value of $x$ : $\sqrt{2x + \sqrt{2x + \sqrt{2x + \dots}}} = 8$ .
12	Solve for $a$ : $9^{a+4} = 81^{2a-3}$
13	Find the value of $x$ : $4 = 1 + \frac{\sqrt{3x}}{1 + \frac{\sqrt{3x}}{1 + \dots}}$
14	If $\frac{7x - 5}{x^2 - 2x - 3} = \frac{A}{x + 1} + \frac{B}{x - 3}$ , find $B - A$ .
15	For what values of $x$ is: $\frac{x - 1}{x + 2} < \frac{x + 2}{x - 1}$ ?

**2010 – 2011 Log1 Contest Round 2**  
**Alpha Equations and Inequalities**

Name: \_\_\_\_\_

4 points each		
1	Solve for x: $5(x - 2) - (x - 4) = 3(x - 1) - 2(x + 1)$	
2	Bertha has a strange farm that only has zebras and ostriches. When she looks out to the field she counts 35 heads and 102 legs. How many zebras does she have?	
3	What is the sum of the roots of $x(2 - 3x)(4x^2 - 3x + 5) = 0$ ?	
4	What is the discriminant of the equation $5x^2 - 9x + 3 = 0$ ?	
5	Find $(f \circ g)(-3)$ if $f(x) = \sqrt{x + 16}$ and $g(x) = 3x^2 - 2x + 15$ .	

5 points each		
6	How many integers are solutions to $ 2x + 1  \leq 5$ ?	
7	If $y$ is inversely proportional to the square of $x$ , and $y = 1/2$ when $x = 10$ , then what is $x$ , greater than 0, when $y = 1/8$ ?	
8	Find the inverse of $y = \sqrt{2 - 7x}$ , when $x \leq \frac{2}{7}$ .	
9	Solve for x: $\log_2 x + \log_4 x^2 = 8$	
10	What is the equation of a line perpendicular to $3x - 2y = 12$ that goes through the point $(-2, 4)$ ? State answer in the form of $Ax + By = C$ .	

6 points each		
11	Find the value of x: $\sqrt{2x + \sqrt{2x + \sqrt{2x + \dots}}} = 8$ .	
12	Solve for a: $9^{a+4} = 81^{2a-3}$	
13	Find the value of x: $4 = 1 + \frac{\sqrt{3x}}{1 + \frac{\sqrt{3x}}{1 + \dots}}$	
14	Find the area of the ellipse: $9x^2 + 4y^2 - 18x - 16y - 11 = 0$	
15	For what values of x is: $\frac{x-1}{x+2} < \frac{x+2}{x-1}$ ?	

**2010 – 2011 Log1 Contest Round 2**  
**Mu Equations and Inequalities**

Name: \_\_\_\_\_

4 points each		
1	Solve for x: $5(x - 2) - (x - 4) = 3(x - 1) - 2(x + 1)$	
2	Bertha has a strange farm that only has zebras and ostriches. When she looks out to the field she counts 35 heads and 102 legs. How many zebras does she have?	
3	What is the sum of the roots of $x(2 - 3x)(4x^2 - 3x + 5) = 0$ ?	
4	What is the discriminant of the equation $5x^2 - 9x + 3 = 0$ ?	
5	If the height of a ball at any time t, in seconds, between 0 and 3 seconds is given by: $h(t) = -16t^2 + 25t + 600$ , when is the speed of the ball equal to zero over this interval?	

5 points each		
6	How many integers are solutions to $ 2x + 1  \leq 5$ ?	
7	If y is inversely proportional to the square of x, and $y = 1/2$ when $x = 10$ , then what is x, greater than 0, when $y = 1/8$ ?	
8	Find the inverse of $y = \sqrt{2 - 7x}$ , when $x \leq \frac{2}{7}$ .	
9	Solve for x: $\log_2 x + \log_4 x^2 = 8$	
10	What is the equation of the line tangent to the curve $x^2 + y^2 = 5$ at the point $(-2, 1)$ ? State answer in the form of $Ax + By = C$ .	

6 points each		
11	Find the value of x: $\sqrt{2x + \sqrt{2x + \sqrt{2x + \dots}}} = 8$ .	
12	Solve for a: $9^{a+4} = 81^{2a-3}$	
13	Find the value of x: $4 = 1 + \frac{\sqrt{3x}}{1 + \frac{\sqrt{3x}}{1 + \dots}}$	
14	Find the area of the ellipse: $9x^2 + 4y^2 - 18x - 16y - 11 = 0$	
15	Solve for t, if $\lim_{x \rightarrow t} \frac{e^x - e^t}{x - t} = 5$ .	

**2010 – 2011 Log1 Contest Round 2**  
**Theta Equations and Inequalities**

Name: \_\_\_\_\_

4 points each		
1	Solve for x: $5(x - 2) - (x - 4) = 3(x - 1) - 2(x + 1)$	$[x=] \frac{1}{3}$
2	Bertha has a strange farm that only has zebras and ostriches. When she looks out to the field she counts 35 heads and 102 legs. How many zebras does she have?	16
3	What is the sum of the roots of $x(2 - 3x)(4x^2 - 3x + 5) = 0$ ?	$\frac{17}{12}$
4	One solution to the equation $x^2 - 3x + 1 = 0$ is $\frac{3 - \sqrt{5}}{2}$ , what is the other solution?	$\frac{3 + \sqrt{5}}{2}$
5	Find $(f \circ g)(-3)$ if $f(x) = \sqrt{x + 16}$ and $g(x) = 3x^2 - 2x + 15$ .	8

5 points each		
6	How many integers are solutions to $ 2x + 1  \leq 5$ ?	6
7	If $y$ is inversely proportional to the square of $x$ , and $y = 1/2$ when $x = 10$ , then what is $x$ , greater than 0, when $y = 1/8$ ?	$[x=] 20$
8	Find the inverse of $y = \sqrt{2 - 7x}$ , when $x \leq \frac{2}{7}$ .	$y = \frac{2 - x^2}{7}$
9	If $9^x = 7$ and $7^y = 3$ , what is $xy$ ?	1/2
10	What is the equation of a line perpendicular to $3x - 2y = 12$ that goes through the point $(-2, 4)$ ? State answer in the form of $Ax + By = C$ .	$2x + 3y = 8$

6 points each		
11	Find the value of $x$ : $\sqrt{2x + \sqrt{2x + \sqrt{2x + \dots}}} = 8$ ?	$[x=] 28$
12	Solve for $a$ : $9^{a+4} = 81^{2a-3}$	$[a=] \frac{10}{3}$
13	Find the value of $x$ : $4 = 1 + \frac{\sqrt{3x}}{1 + \frac{\sqrt{3x}}{1 + \dots}}$	$[x=] 48$
14	If $\frac{7x - 5}{x^2 - 2x - 3} = \frac{A}{x + 1} + \frac{B}{x - 3}$ , find $B - A$ .	1
15	For what values of $x$ is: $\frac{x - 1}{x + 2} < \frac{x + 2}{x - 1}$ ?	$x > 1$ union $-2 < x < -1/2$

**2010 – 2011 Log1 Contest Round 2**  
**Alpha Equations and Inequalities**

Name: \_\_\_\_\_

4 points each		
1	Solve for x: $5(x - 2) - (x - 4) = 3(x - 1) - 2(x + 1)$	$[x=] \frac{1}{3}$
2	Bertha has a strange farm that only has zebras and ostriches. When she looks out to the field she counts 35 heads and 102 legs. How many zebras does she have?	16
3	What is the sum of the roots of $x(2 - 3x)(4x^2 - 3x + 5) = 0$ ?	$\frac{17}{12}$
4	What is the discriminant of the equation $5x^2 - 9x + 3 = 0$ ?	21
5	Find $(f \circ g)(-3)$ if $f(x) = \sqrt{x + 16}$ and $g(x) = 3x^2 - 2x + 15$ .	8

5 points each		
6	How many integers are solutions to $ 2x + 1  \leq 5$ ?	6
7	If $y$ is inversely proportional to the square of $x$ , and $y = 1/2$ when $x = 10$ , then what is $x$ , greater than 0, when $y = 1/8$ ?	$[x=] 20$
8	Find the inverse of $y = \sqrt{2 - 7x}$ , when $x \leq \frac{2}{7}$ .	$y = \frac{2 - x^2}{7}$
9	Solve for x: $\log_2 x + \log_4 x^2 = 8$	16
10	What is the equation of a line perpendicular to $3x - 2y = 12$ that goes through the point $(-2, 4)$ ? State answer in the form of $Ax + By = C$ .	$2x + 3y = 8$

6 points each		
11	Find the value of x: $\sqrt{2x + \sqrt{2x + \sqrt{2x + \dots}}} = 8$ ?	$[x=] 28$
12	Solve for a: $9^{a+4} = 81^{2a-3}$	$[a=] \frac{10}{3}$
13	Find the value of x: $4 = 1 + \frac{\sqrt{3x}}{1 + \frac{\sqrt{3x}}{1 + \dots}}$	$[x=] 48$
14	Find the area of the ellipse: $9x^2 + 4y^2 - 18x - 16y - 11 = 0$	$6\pi$
15	For what values of x is: $\frac{x-1}{x+2} < \frac{x+2}{x-1}$ ?	$x > 1$ union $-2 < x < -1/2$

**2010 – 2011 Log1 Contest Round 2**  
**Mu Equations and Inequalities**

Name: \_\_\_\_\_

4 points each		
1	Solve for x: $5(x - 2) - (x - 4) = 3(x - 1) - 2(x + 1)$	$[x=] \frac{1}{3}$
2	Bertha has a strange farm that only has zebras and ostriches. When she looks out to the field she counts 35 heads and 102 legs. How many zebras does she have?	16
3	What is the sum of the roots of $x(2 - 3x)(4x^2 - 3x + 5) = 0$ ?	$\frac{17}{12}$
4	What is the discriminant of the equation $5x^2 - 9x + 3 = 0$ ?	21
5	If the height of a ball at any time t, in seconds, between 0 and 3 seconds is given by: $h(t) = -16t^2 + 25t + 600$ , when is the speed of the ball equal to zero over this interval?	$\frac{25}{32}$ [secs]

5 points each		
6	How many integers are solutions to $ 2x + 1  \leq 5$ ?	6
7	If y is inversely proportional to the square of x, and $y = 1/2$ when $x = 10$ , then what is x, greater than 0, when $y = 1/8$ ?	$[x=] 20$
8	Find the inverse of $y = \sqrt{2 - 7x}$ , when $x \leq \frac{2}{7}$ .	$y = \frac{2 - x^2}{7}$
9	Solve for x: $\log_2 x + \log_4 x^2 = 8$	16
10	What is the equation of the line tangent to the curve $x^2 + y^2 = 5$ at the point $(-2, 1)$ ? State answer in the form of $Ax + By = C$ .	$2x - y = -5$

6 points each		
11	Find the value of x: $\sqrt{2x + \sqrt{2x + \sqrt{2x + \dots}}} = 8$ ?	$[x=] 28$
12	Solve for a: $9^{a+4} = 81^{2a-3}$	$[a=] \frac{10}{3}$
13	Find the value of x: $4 = 1 + \frac{\sqrt{3x}}{1 + \frac{\sqrt{3x}}{1 + \dots}}$	$[x=] 48$
14	Find the area of the ellipse: $9x^2 + 4y^2 - 18x - 16y - 11 = 0$	$6\pi$
15	Solve for t, if $\lim_{x \rightarrow t} \frac{e^x - e^t}{x - t} = 5$ .	$\ln 5$

**2010 – 2011 Log1 Contest Round 2**  
**Equations and Inequalities Solutions**

Mu	Al	Th	Solution
1	1	1	$5(x - 2) - (x - 4) = 3(x - 1) - 2(x + 1)$ $5x - 10 - x + 4 = 3x - 3 - 2x - 2$ $3x = 1$ $x = \frac{1}{3}$
2	2	2	<p>A zebra has one head and four legs, and an ostrich has one head and two legs, so the equation is set up as:</p> $\begin{cases} Z + O = 35 \\ 4Z + 2O = 102 \end{cases}$ <p>Any method can be used to solve for Z.</p> $\begin{cases} (-2)(Z + O) = (-2)(35) \\ 4Z + 2O = 102 \end{cases}$ $\begin{cases} -2Z - 2O = -70 \\ 4Z + 2O = 102 \end{cases}$ $2Z = 38$ $Z = 16$
3	3	3	$x(2 - 3x)(4x^2 + 5 - 3x) =$ $(2x - 3x^2)(4x^2 - 3x + 5) =$ $8x^3 - 6x^2 + 10x - 12x^4 + 9x^3 - 15x^2 =$ $-12x^4 + 17x^3 - 21x^2 + 10x$ <p>Sum of roots is <math>-\frac{b}{a}</math> so the answer is <math>\frac{17}{12}</math>.</p> <p>Of course, two of the roots are clearly 0 and <math>\frac{2}{3}</math>. So we could just add the sum of the roots from the quadratic term <math>\frac{3}{4}</math>.</p>
4	4		Discriminant is $b^2 - 4ac$ , so in this case, it is $(-9)^2 - 4(5)(3) = 81 - 60 = 21$
		4	The product of the solutions must be 1, the other solution will just invert the sign in front of the square root. $\frac{3 + \sqrt{5}}{2}$
5			<p>One can solve this as the vertex of a parabola, or simply find the speed function.</p> $s(t) = h'(t) = -32t + 25 = 0$ $t = \frac{25}{32}$
	5	5	$(f \circ g)(-3) = f(g(-3)) =$ $\sqrt{(3(-3))^2 - 2(-3) + 15} + 16 =$ $\sqrt{27 + 6 + 15 + 16} = \sqrt{64} = 8$

6	6	6	$ 2x+1  \leq 5$ $-5 \leq 2x+1 \leq 5$ $-6 \leq 2x \leq 4$ $-3 \leq x \leq 2$  $x: \{-3, -2, -1, 0, 1, 2\}$ There are six integers in the solution set.
7	7	7	$y = \frac{k}{x^2}$  $\frac{1}{2} = \frac{k}{10^2}$ $50 = k$  $\frac{1}{8} = \frac{50}{x^2}$ $x^2 = 400$ $x = 20$
8	8	8	$x = \sqrt{2-7y}$ $x^2 = (\sqrt{2-7y})^2$ $x^2 = 2-7y$ $7y = 2-x^2$ $y = \frac{2-x^2}{7}$
9	9		$\log_2 x + \log_4 x^2 = 8$ $\frac{\log_4 x}{\log_4 2} + \frac{\log_4 x^2}{\log_4 4} = 8$ $\frac{\log_4 x}{\frac{1}{2}} + \log_4 x^2 = 8$ $2\log_4 x + \log_4 x^2 = 8$ $2\log_4 x + 2\log_4 x = 8$ $4\log_4 x = 8$ $\log_4 x = 2$ $x = 4^2 = 16$
		9	$3^1 = 7^y = (9^x)^y = ((3^2)^x)^y = 3^{2xy}$  $1 = 2xy$ $xy = 1/2$
10			The curve is just a circle centered at the origin, so the tangent line will be perpendicular to the line from the origin to (-2,1) which has slope -1/2. The line will have slope 2, thus $y=2x+5$ . Re-arranging the terms $2x-y = -5$ . $-2x+y=5$ is also acceptable.



10	10	10	$3x - 2y = 12 \Rightarrow y = \frac{3x - 12}{2}$ <p>Slope of perpendicular line is <math>-\frac{2}{3}</math>.</p> <p>Equation of line with slope <math>-\frac{2}{3}</math> through <math>(-2, 4)</math>:</p> $y - 4 = -\frac{2}{3}(x + 2)$ $y = -\frac{2}{3}x - \frac{4}{3} + 4$ $3y = -2x - 4 + 12$ $2x + 3y = 8$
11	11	11	$(\sqrt{2x} + \sqrt{2x} + \sqrt{2x} + \dots)^2 = 8^2$ $2x + \sqrt{2x} + \sqrt{2x} + \dots = 64$ $2x + 8 = 64$ $2x = 56$ $x = 28$
12	12	12	$3^{2(a+4)} = 3^{4(2a-3)}$ $2(a+4) = 4(2a-3)$ $2a+8 = 8a-12$ $20 = 6a$ $a = \frac{10}{3}$
13	13	13	$4 = 1 + \frac{\sqrt{3x}}{1 + \frac{\sqrt{3x}}{1 + \dots}}$ $4(4) = 4\left(1 + \frac{\sqrt{3x}}{4}\right)$ $16 = 4 + \sqrt{3x}$ $12^2 = (\sqrt{3x})^2$ $144 = 3x$ $x = 48$
14	14		<p>Rearrange equation and complete the square twice to get standard form of ellipse:</p> $9x^2 - 18x + 4y^2 - 16y = 11$ $9(x^2 - 2x) + 4(y^2 - 4y) = 11$ $9(x-1)^2 + 4(y-2)^2 = 11 + 9 + 16$ $9(x-1)^2 + 4(y-2)^2 = 36$ $\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$ <p>Area of ellipse is <math>r_1 r_2 \pi</math> where <math>r_1</math> is one-half the length of one axis and <math>r_2</math> is one-half the length of the other axis, so <math>A = \sqrt{4} \sqrt{9} \pi = 6\pi</math></p>

		14	$\frac{7x-5}{x^2-2x-3} = \frac{A(x-3)}{x^2-2x-3} + \frac{B(x+1)}{x^2-2x-3}$ $7x-5 = A(x-3) + B(x+1)$ <p>Let <math>x = 3</math></p> $7(3) - 5 = A(3-3) + B(3+1)$ $16 = A(0) + 4B$ $B = 4$ <p>Let <math>x = -1</math></p> $7(-1) - 5 = A(-1-3) + B(-1+1)$ $-7 - 5 = -4A + B(0)$ $A = 3$ $B - A = 1$
15			The limit is simply the derivative of $e^x$ evaluated at $t$ . The equation becomes $e^t=5$ with solution $\ln 5$ .
	15	15	<p>The temptation to cross-multiply must be avoided. Instead subtract one side from the other and simplify.</p> $\frac{x+2}{x-1} - \frac{x-1}{x+2} > 0$ $\frac{3(2x+1)}{(x-1)(x+2)} > 0$ <p>Either all three terms are positive or exactly two are negative. So, either <math>x &gt; 1</math> (all positive) or <math>-2 &lt; x &lt; -1/2</math> which makes <math>x-1</math> and <math>2x+1</math> negative.</p>