2010 – 2011 Log1 Contest Round 1
Theta Algebra and Functions

Name: _____

	4 points each	
1	What is the value of x if: $3x+4=25$?	
2	If an 8-slice pizza is made with 3 pounds of cheese that is distributed evenly over the entire pizza, then how many pounds of cheese are on a slice and a half of pizza? Express your answer as a fraction.	
3	If $s=5$ and $t=2$, then what is $st^2-(t-s)$?	
4	Find x: $x+6-3(x-1)=5x+4$.	
5	If $f(x)=3x+1$ and $g(x)=x^2-x-1$ and $h(x)=f(x)g(x)$, then what is $h(3)$?	

	5 points each	
6	Evaluate: $\sqrt{\sqrt{3}\cdot\sqrt{4(7)-1}}$.	
7	What is the equation in slope-intercept form of the line that is perpendicular to the line $y=2x-5$ and contains the point $(6,4)$?	
8	Let $f(x)=f(x-1)+f(x-2)$ for all integers x. If $f(1)=f(2)=1$, then what is the value of $f(10)$?	
9	What is the value(s) of x if: $6e^{2x}+11e^{x}-10=0$?	
10	Simplify completely (without negative exponents): $\frac{a^3b^{-5}(c^2)^3a^{-5}}{a^4bc^4}$	

	6 points each	
11	Solve for x : $\sqrt{43+\sqrt{43-\sqrt{43+\sqrt{43-x}}}}=x$	
12	How many integer pairs (x,y) are solutions to the following system of inequalities? $x-2y\geq -2$ $3x-y\leq 9$ $y\geq 0$	
13	What is the remainder when $1!+2!+3!+\ldots+2010!$ is divided by 15?	
14	At what values of x do the graphs of $x^2+y^2=1$ and $y=x^2$ intersect?	
15	How many integer solutions are there to the following inequality? $2x-1<\lvert x+1\rvert<3x+2$	

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9	What is the value(s) of x if: $6e^{2x}+11e^{x}-10=0$?	
10	For how many integers, n, between 0 and 9 inclusive will $f^{-1}\left(x\right)$ exist if $f\left(x\right)=nx^n$ for all real values of x?	

	6 points each	
11	Solve for x : $\sqrt{43+\sqrt{43-\sqrt{43+\sqrt{43-x}}}}=x$	
12	How many integer pairs (x,y) are solutions to the following system of inequalities? $x-2y\geq -2$ $3x-y\leq 9$ $y\geq 0$	
13	What is the remainder when $1!+2!+3!+\ldots+2010!$ is divided by 15?	
14	At what values of x do the graphs of $x^2+y^2=1$ and $y=x^2$ intersect?	
15	What is the value of $x^4+\frac{1}{x^4}$ given $x+\frac{1}{x}=5$?	

2010 – 2011 Log1 Contest Round 1
Mu Algebra and Functions

Name: _____

	4 points each	
1	What is the value of x if: $3x+4=25$?	
2	If an 8-slice pizza is made with 3 pounds of cheese that is distributed evenly over the entire pizza, then how many pounds of cheese are on a slice and a half of pizza? Express your answer as a fraction.	
3	If $s=5$ and $t=2$, then what is $st^2-(t-s)$?	
4	Evaluate: $\lim_{x\to 2}\left(x^2+3x+2\right)$	
5	What is the sum of the coefficients of the binomial expansion: $\left(2x+5y\right)^3$	

	5 points each	
6	Evaluate: $\sqrt{\sqrt{3}\cdot\sqrt{4(7)-1}}$.	
7	What is the equation in slope-intercept form of the line that is perpendicular to the line $y=2x-5$ and contains the point $\left(6,4\right)$?	
8	Let $f(x)=f(x-1)+f(x-2)$ for all integers *x*. If $f(1)=f(2)=1$, then what is the value of $f(10)$?	
9	What is the global maximum, (*x*,*y*), of the graph: $f\left(x\right)=-3x^4+8x^3+30x^2-72x+72$?	
10	For how many integers, *n*, between 0 and 9 inclusive will $f^{-1}\left(x\right)$ exist if $f\left(x\right)=nx^n$ for all real values of *x*?	

	6 points each	
11	Solve for x : $\sqrt{43+\sqrt{43-\sqrt{43+\sqrt{43-x}}}}=x$	
12	How many integer pairs (*x*,*y*) are solutions to the following system of inequalities? $x-2y\geq -2$ $3x-y\leq 9$ $y\geq 0$	
13	What is the remainder when $1!+2!+3!+\ldots+2010!$ is divided by 15?	
14	What is the $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$?	
15	What is the value of $x^4+\frac{1}{x^4}$ given $x+\frac{1}{x}=5$?	

2010 – 2011 Log1 Contest Round 1
Theta Algebra and Functions

Name: _____

4 points each		
1	What is the value of x if: $3x+4=25$?	7
2	If an 8-slice pizza is made with 3 pounds of cheese that is distributed evenly over the entire pizza, then how many pounds of cheese are on a slice and a half of pizza? Express your answer as a fraction.	$\frac{9}{16}$ [pounds]
3	If $s=5$ and $t=2$, then what is $st^2-(t-s)$?	23
4	Find x: $x+6-3(x-1)=5x+4$.	5/7
5	If $f(x)=3x+1$ and $g(x)=x^2-x-1$ and $h(x)=f(x)g(x)$, then what is $h(3)$?	50

5 points each		
6	Evaluate: $\sqrt{\sqrt{3}\cdot\sqrt{4(7)-1}}$.	3
7	What is the equation in slope-intercept form of the line that is perpendicular to the line $y=2x-5$ and contains the point $(6,4)$?	$y=-\frac{1}{2}x+7$
8	Let $f(x)=f(x-1)+f(x-2)$ for all integers x. If $f(1)=f(2)=1$, then what is the value of $f(10)$?	55
9	What is the value(s) of x if: $6e^{2x}+11e^{x}-10=0$?	$x=\ln\left(\frac{2}{3}\right)$
10	Simplify completely (without negative exponents): $\frac{a^3b^{-5}(c^2)^3a^{-5}}{a^4bc^4}$	$\frac{c^2}{a^6b^6}$

6 points each		
11	Solve for x : $\sqrt{43+\sqrt{43-\sqrt{43+\sqrt{43-x}}}}=x$	$x=7$
12	How many integer pairs (x,y) are solutions to the following system of inequalities? $x-2y\geq -2$ $3x-y\leq 9$ $y\geq 0$	13 [pairs]
13	What is the remainder when $1!+2!+3!+\ldots+2010!$ is divided by 15?	3
14	At what values of x do the graphs of $x^2+y^2=1$ and $y=x^2$ intersect?	$x=\pm\sqrt{\frac{\sqrt{5}-1}{2}}$
15	How many integer solutions are there to the following inequality? $2x-1<\lvert x+1\rvert<3x+2$	2 [solutions]

2010 – 2011 Log1 Contest Round 1
Alpha Algebra and Functions

Name: _____

	4 points each	
1	What is the value of x if: $3x+4=25$?	7
2	If an 8-slice pizza is made with 3 pounds of cheese that is distributed evenly over the entire pizza, then how many pounds of cheese are on a slice and a half of pizza? Express your answer as a fraction.	$\frac{9}{16}$ [pounds]
3	If $s=5$ and $t=2$, then what is $st^2-(t-s)$?	23
4	Find x: $x+6-3(x-1)=5x+4$.	5/7
5	What is the sum of the coefficients of the binomial expansion: $\left(2x+5y\right)^3$	343

	5 points each	
6	Evaluate: $\sqrt{\sqrt{3}\cdot\sqrt{4(7)-1}}$.	3
7	What is the equation in slope-intercept form of the line that is perpendicular to the line $y=2x-5$ and contains the point $\left(6,4\right)$?	$y=-\frac{1}{2}x+7$
8	Let $f(x)=f(x-1)+f(x-2)$ for all integers x. If $f(1)=f(2)=1$, then what is the value of $f(10)$?	55
9	What is the value(s) of x if: $6e^{2x}+11e^{x}-10=0$?	$x=\ln\left(\frac{2}{3}\right)$
10	For how many integers, n, between 0 and 9 inclusive will $f^{-1}\left(x\right)$ exist if $f\left(x\right)=nx^n$ for all real values of x?	5 [integers]

	6 points each	
11	Solve for x : $\sqrt{43+\sqrt{43-\sqrt{43+\sqrt{43-x}}}}=x$	$x=7$
12	How many integer pairs (x,y) are solutions to the following system of inequalities? $x-2y\geq-2$ $3x-y\leq9$ $y\geq0$	13 [pairs]
13	What is the remainder when $1!+2!+3!+\ldots+2010!$ is divided by 15?	3
14	At what values of x do the graphs of $x^2+y^2=1$ and $y=x^2$ intersect?	$x=\pm\sqrt{\frac{\sqrt{5}-1}{2}}$
15	What is the value of $x^4+\frac{1}{x^4}$ given $x+\frac{1}{x}=5$?	527

2010 – 2011 Log1 Contest Round 1
Mu Algebra and Functions

Name: _____

	4 points each	
1	What is the value of x if: $3x+4=25$?	7
2	If an 8-slice pizza is made with 3 pounds of cheese that is distributed evenly over the entire pizza, then how many pounds of cheese are on a slice and a half of pizza? Express your answer as a fraction.	$\frac{9}{16}$ [pounds]
3	If $s=5$ and $t=2$, then what is $st^2-(t-s)$?	23
4	Evaluate: $\lim_{x\to 2}\left(x^2+3x+2\right)$	12
5	What is the sum of the coefficients of the binomial expansion: $\left(2x+5y\right)^3$	343

	5 points each	
6	Evaluate: $\sqrt{\sqrt{3}\cdot\sqrt{4(7)-1}}$.	3
7	What is the equation in slope-intercept form of the line that is perpendicular to the line $y=2x-5$ and contains the point $\left(6,4\right)$?	$y=-\frac{1}{2}x+7$
8	Let $f(x)=f(x-1)+f(x-2)$ for all integers *x*. If $f(1)=f(2)=1$, then what is the value of $f(10)$?	55
9	What is the global maximum, (*x*,*y*), of the graph: $f\left(x\right)=-3x^4+8x^3+30x^2-72x+72$?	$\left(-2,224\right)$
10	For how many integers, *n*, between 0 and 9 inclusive will $f^{-1}\left(x\right)$ exist if $f\left(x\right)=nx^n$ for all real values of *x*?	5 [integers]

	6 points each	
11	Solve for x : $\sqrt{43+\sqrt{43-\sqrt{43+\sqrt{43-x}}}}=x$	$x=7$
12	How many integer pairs (*x*,*y*) are solutions to the following system of inequalities? $x-2y\geq -2$ $3x-y\leq 9$ $y\geq 0$	13 [pairs]
13	What is the remainder when $1!+2!+3!+\ldots+2010!$ is divided by 15?	3
14	What is the $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$?	0
15	What is the value of $x^4+\frac{1}{x^4}$ given $x+\frac{1}{x}=5$?	527

2010 – 2011 Log1 Contest Round 1
Algebra and Functions Solutions

Mu	Al	Th	Solution
1	1	1	Subtract 4 and divide by 3 to get x=7.
2	2	2	Since a slice of pizza is one eighth of the whole pizza and the cheese is spread out evenly, then there are 3/8 pounds of cheese on one slice. 3/8 + 3/16 = 9/16.
3	3	3	PEMDAS. 5(4) – (-3) = 23
	4	4	Combining like terms and solving for x using symbolic manipulation we see that -2x+9=5x+4 or 7x=5, x=5/7.
4			Since the function is continuous, all that is needed is to evaluate the function at x=2. 12.
		5	There is no need to multiply the functions, just multiply f(3)=10 times g(3)=5 to get 50.
5	5		The sum of the coefficients of the binomial expansion can be found by substituting x = y = 1 and evaluating the exponent. Thus seven cubed is 343.
6	6	6	$\sqrt{\sqrt{3}\times\sqrt{4\times7-1}}$ $=\sqrt{\sqrt{3}\times\sqrt{27}}$ $=\sqrt{\sqrt{81}}$ $=\sqrt{9}=3$
7	7	7	The slope of the perpendicular line will be the negative inverse of the original line. Thus the slope is negative one-half. With the given point solve for the y-intercept (7) and write the equation in slope intercept form: $y=-\frac{1}{2}x+7$
8	8	8	Notice that the value of f(x) depends on the previous two values of f(x). Brute forcing this evolves the answer of 55. f(3)=f(1)+f(2)=2, etc.
	9	9	Notice that if we let $y=e^x$, the equation becomes a quadratic. Solving for y yields: $y=-\frac{5}{2},\frac{2}{3}$. But notice that e^x can only be positive so therefore $x=\ln\left(\frac{2}{3}\right)$.
9			First notice that the graph extends into negative infinity once x is not in the neighborhood of f(x)'s roots (graph looks like an 'M'). So the global maximum of the graph occurs when: $f'(x)=0=-12x^3+24x^2+60x-72$. Solving for x yields that x = -2, 1, 3; plugging this back into *f* we obtain a corresponding y = 224 (y=99 when x=3). Thus (-2,224).
		10	$\frac{a^3b^{-5}(c^2)^3a^{-5}}{a^4bc^4}=\frac{a^3c^6}{a^4a^5bb^5c^4}=\frac{c^2}{a^6b^6}$
10	10		For a function to have an inverse, it must pass the horizontal line test: any horizontal line drawn will intersect the function at most 1 point. Notice that this property only holds for odd values of n. Thus there are 5 possible values of n.
11	11	11	If x satisfies $\sqrt{43+\sqrt{43-x}}=x$, then it also satisfies the original equation. Squaring this equation twice yields $x^4-86x^2+x+43(42)=0$ Trying positive factors of 43(42), yields x=7 which can be verified.

12	12	12	Drawing these three lines yields a triangle with vertices (-2,0), (3,0) and (4,3). There are 6 points with y=0, 4 with y=1, 2 with y=2 and 1 with y=3.
13	13	13	Notice that $n!$ for $n \geq 5$ is divisible by 15, so any addition of these values won't contribute to the remainder. So summing up the first four factorials (33) and dividing by 15, we see a remainder of 3.
	14	14	By substituting $y = x^2$ into the equation for a circle we obtain a quadratic equation in terms of y. Solving for y, we get: $y = \frac{-1+\sqrt{5}}{2}$ as y is necessarily positive. Taking the square root gives us: $x = \pm\sqrt{\frac{\sqrt{5}-1}{2}}$.
14			Let $f(x) = \frac{1}{x} - \frac{1}{\sin x} = \frac{\sin x - x}{x \sin x}$ which approaches $0/0$ so use L'Hopital's rule. $f'(x) = \frac{\cos x - 1}{\sin x + x \cos x}$ which also approaches $0/0$, so $f''(x) = \frac{-\sin x}{2\cos x - x \sin x}$ which approaches $0/2 = 0$.
		15	Graphing these three curves, we see that only when x = 0 or 1 will $2x - 1 < \lvert x+1 \rvert < 3x+2$ hold true. Therefore 2 integer solutions.
15	15		$\left(x + \frac{1}{x}\right)^4 = 625 = x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$ $\left(x + \frac{1}{x}\right)^2 = 25 = x^2 + 2 + \frac{1}{x^2}$ Combining, we get $$619 = x^4 + \frac{1}{x^4} + 4(23)$$ so the answer is 619-92=527.