2009 – 2010 Log1 Contest Round 2
Theta Geometry

Name: _____

4 points each		
1	What is the area of an isosceles right triangle with legs of length 3?	
2	What is perimeter of a regular heptagon with sides of length 8?	
3	Matt built a 3 foot concrete walkway around his circular garden. If his garden is 6 feet across in diameter, what is the area of the walkway in square feet?	
4	How many diagonals are there in a regular nonagon?	
5	In a regular quadrilateral, what is the ratio of the diagonal length to its side length?	

5 points each		
6	How many points of intersection are possible in a plane figure that consists of three congruent but distinct circles?	
7	What is the measure of the smaller angle in degrees created by the two hands of an analog clock face at 10:10?	
8	What is the area of a triangle with sides of length 7, 8, and 9?	
9	An ant walks an elliptical path that at its widest (major axis) measures 18 inches and its minor axis is 14 inches. What is the area of the region that the ant makes?	
10	A sphere that has a radius of 4 cm is cut by a plane that is 1 cm from the center of the sphere at its closest point. What is the area of the circle created by the intersection?	

6 points each		
11	What is the positive difference between the longest and shortest diagonal that can be formed in a regular hexagon of side length 1?	
12	A smaller hexagon is inscribed in a larger regular hexagon by connecting the midpoints of each side. The smaller hexagon has side length 2, what is the area of the larger hexagon?	
13	A conical funnel is placed point down and a spherical marble with radius 4 mm is dropped in touching the cone in a circle. A larger marble with radius 6 mm is then dropped in and it touches the first marble and is tangent to the cone in a circle. A third, still larger, marble is then placed in the cone, touching the second one and tangent to the cone. What is the radius of the third marble, in mm?	
14	A point x is in the interior of a square ABCD. The point is $\sqrt{10}$ units from vertices A and C and $\sqrt{2}$ units from vertex B. How many units is the point x from vertex D?	
15	A circle has equation $12x - 2y^2 + 28y - 2x^2 = 84$. What is its area?	

2009 – 2010 Log1 Contest Round 2
Alpha Geometry

Name: _____

4 points each		
1	What is the area of an isosceles right triangle with legs of length 3?	
2	What is perimeter of a regular heptagon with sides of length 8?	
3	Matt built a 3 foot concrete walkway around his circular garden. If his garden is 6 feet across in diameter, what is the area of the walkway in square feet?	
4	How many diagonals are there in a regular icosagon?	
5	In a regular quadrilateral, what is the ratio of the diagonal length to its side length?	

5 points each		
6	How many points of intersection are possible in a plane figure that consists of three congruent but distinct circles?	
7	What is the measure of the smaller angle in degrees created by the two hands of an analog clock face at 10:10?	
8	What is the area of a triangle with sides of length 7, 8, and 9?	
9	An ant walks an elliptical path that at its widest (major axis) measures 18 inches and its minor axis is 14 inches. What is the area of the region that the ant makes?	
10	A sphere that has a radius of 4 cm is cut by a plane that is 1 cm from the center of the sphere at its closest point. What is the area of the circle created by the intersection?	

6 points each		
11	What is the length of the longest diagonal that can be formed in a regular octagon of side length 1?	
12	A smaller hexagon is inscribed in a larger regular hexagon by connecting the midpoints of each side. The smaller hexagon has side length 2, what is the area of the larger hexagon?	
13	A conical funnel is placed point down and a spherical marble with radius 4 mm is dropped in touching the cone in a circle. A larger marble with radius 6 mm is then dropped in and it touches the first marble and is tangent to the cone in a circle. A third, still larger, marble is then placed in the cone, touching the second one and tangent to the cone. What is the radius of the third marble, in mm?	
14	A point x is in the interior of a square ABCD. The point is $\sqrt{10}$ units from vertices A and C and $\sqrt{2}$ units from vertex B. How many units is the point x from vertex D?	
15	A circle has equation $12x - 2y^2 + 28y - 2x^2 = 84$. What is its area?	

2009 – 2010 Log1 Contest Round 2
Mu Geometry

Name: _____

4 points each		
1	What is the area of an isosceles right triangle with legs of length 3?	
2	What is perimeter of a regular heptagon with sides of length 8?	
3	Matt built a 3 foot concrete walkway around his circular garden. If his garden is 6 feet across in diameter, what is the area of the walkway in square feet?	
4	How many diagonals are there in a regular icosagon?	
5	What is the ratio of the length of the largest diagonal to the side of a regular hexagon?	

5 points each		
6	How many points of intersection are possible in a plane figure that consists of three congruent but distinct circles?	
7	What is the measure of the smaller angle in degrees created by the two hands of an analog clock face at 10:10?	
8	What is the area of a triangle with sides of length 7, 8, and 9?	
9	An ant walks an elliptical path that at its widest (major axis) measures 18 inches and its minor axis is 14 inches. What is the area of the region that the ant makes?	
10	The lateral surface area of a right circular cone is three times the area of the base. If the base area is 36π , what is the volume of the cone?	

6 points each		
11	What is the length of the longest diagonal that can be formed in a regular octagon of side length 1?	
12	A smaller hexagon is inscribed in a larger regular hexagon by connecting the midpoints of each side. The smaller hexagon has side length 2, what is the area of the larger hexagon?	
13	A conical funnel is placed point down and a spherical marble with radius 4 mm is dropped in touching the cone in a circle. A larger marble with radius 6 mm is then dropped in and it touches the first marble and is tangent to the cone in a circle. A third, still larger, marble is then placed in the cone, touching the second one and tangent to the cone. What is the radius of the third marble, in mm?	
14	What is the area of the spherical triangle drawn on a sphere of radius 2 cm with angles of measure, in radians, of $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}$.	
15	The curves y = x and $y = x^2$ are defined on the interval [0,2]. What is the area between these curves on that interval?	

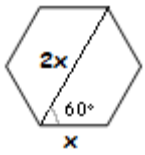
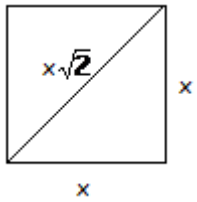
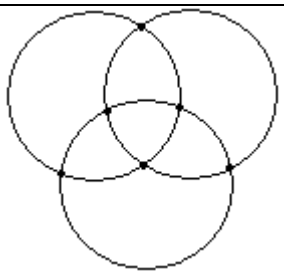
2009 – 2010 Log1 Contest Round 2
Geometry Answers

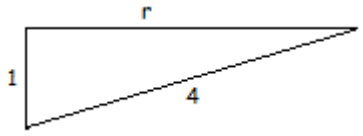
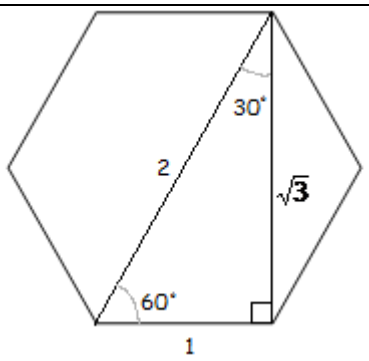
Theta Answers	
1	$\frac{9}{2}$
2	56
3	27π [ft^2]
4	27
5	$\sqrt{2}$ or $\sqrt{2}:1$
6	6
7	115 [°]
8	$12\sqrt{5}$
9	63π
10	15π
11	$2-\sqrt{3}$
12	$8\sqrt{3}$
13	9 [mm]
14	$3\sqrt{2}$
15	16π

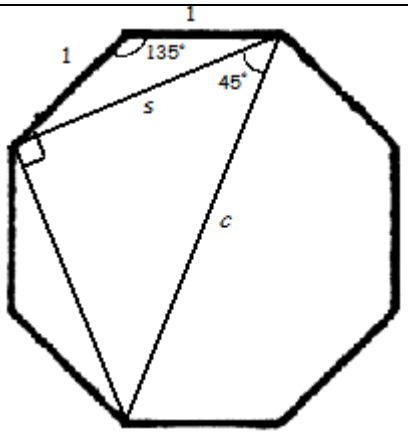
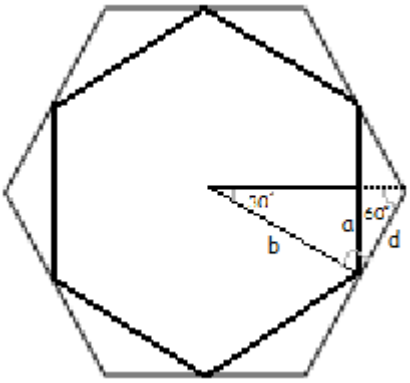
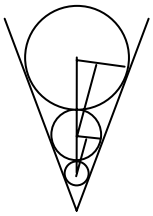
Alpha Answers	
1	$\frac{9}{2}$
2	56
3	27π [ft^2]
4	170
5	$\sqrt{2}$ or $\sqrt{2}:1$
6	6
7	115 [°]
8	$12\sqrt{5}$
9	63π
10	15π
11	$\sqrt{4+2\sqrt{2}}$
12	$8\sqrt{3}$
13	9 [mm]
14	$3\sqrt{2}$
15	16π

Mu Answers	
1	$\frac{9}{2}$
2	56
3	27π [ft^2]
4	170
5	2 or 2 : 1
6	6
7	115 [°]
8	$12\sqrt{5}$
9	63π
10	$144\sqrt{2}\pi$
11	$\sqrt{4+2\sqrt{2}}$
12	$8\sqrt{3}$
13	9 [mm]
14	2π [*sq.cm*]
15	1

2009 – 2010 Log1 Contest Round 2
Geometry Solutions

Mu	Al	Th	Solution
1	1	1	$A = \frac{1}{2}bh$ $= \frac{1}{2}(3)(3)$ $= \frac{9}{2}$
2	2	2	8(7) = 56
3	3	3	Area of walkway = Area of larger circle – Area of Garden $= \pi(3+3)^2 - \pi(3)^2$ $= 36\pi - 9\pi$ $= 27\pi$
		4	No. of diagonals $= \frac{n(n-3)}{2}$. Each of the n vertices can be connected to (n-3) others but this would double count each diagonal. $= \frac{9(6)}{2}$ $= 27$
4	4		$\frac{20(17)}{2} = 170$
5			 $\frac{2x}{x} = 2$ $2x : x = 2 : 1$
	5	5	 $\frac{x\sqrt{2}}{x} = \sqrt{2}$ $x\sqrt{2} : x = \sqrt{2} : 1$
6	6	6	
7	7	7	Angle at 10:10 (without the added movement of the hour hand) = 120° Movement of hour hand $= \frac{10}{60}(30) = 5°$ 120 – 5 = 115°

8	8	8	Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ A is the area, s is the semi-perimeter, and the side lengths are a, b, and c. $s = \frac{7+8+9}{2} = 12$ $A = \sqrt{12(12-7)(12-8)(12-9)}$ $= 12\sqrt{5}$
9	9	9	A = πab a = minor axis b = major axis $A = 9(7)\pi$ $= 63\pi$
	10	10	 $r = \sqrt{4^2 - 1}$ $= \sqrt{15}$ $A = (\sqrt{15})^2\pi = 15\pi$
10			Lateral area $= \pi r l$ Base $= \pi r^2 = 36\pi$ $\therefore r = 6$ $3(36\pi) = \pi 6 l$ $108 = 6l$ $l = 18$ $h = \sqrt{l^2 - r^2} = \sqrt{324 - 36}$ $= \sqrt{(3\sqrt{42})^2 - (\sqrt{42})^2}$ $= 12\sqrt{2}$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(6)^2(12\sqrt{2})$ $= 144\sqrt{2}\pi$
		11	 longest – shortest $= 2 - \sqrt{3}$

11	11		 $s^2 = 1^2 + 1^2 - 2(1)(1)\cos 135°$ $s = \sqrt{2+\sqrt{2}}$ $c = s\sqrt{2}$ $= \sqrt{4+2\sqrt{2}}$
12	12	12	 The apothem of the larger hexagon is b=2. This makes the side length of the larger hexagon $4/\sqrt{3}$. The area of the hexagon (sum of 6 triangles) is then: $A = (apothem)(perimeter)/2 = (2)(6)(4/\sqrt{3})/2$ $= 8\sqrt{3}$
13	13	13	The radii of the marbles form a geometric sequence. This can be seen by the considering the drawing. The small triangles are similar. If the three marbles have radii: r_1, r_2, r_3 , we have $\frac{r_2 + r_1}{r_2 - r_1} = \frac{r_3 + r_2}{r_3 - r_2}$ Cross multiply $r_2^2 = r_1 r_2$. Thus,  and simplify gives: the third marble will be 6 (6/4) = 9 mm.

	14	14	Since the point is equidistant from A and C, it must be on the BD diagonal. From that, the side length can be seen to 4 and the missing length calculated. What follows is a more general solution. Let v and w be the horizontal and vertical displacement of the point x from vertex A. Let the distance from the point x to vertex D be d, the side length s, and we get the following 4 equations using the Pythagorean Formula: $v^2 + w^2 = (\sqrt{10})^2 = 10$ $v^2 + (s-w)^2 = (\sqrt{2})^2 = 2$ $(s-v)^2 + (s-w)^2 = (\sqrt{10})^2 = 10$ $(s-v)^2 + w^2 = d^2$ Subtract the second equation from the first and the third from the fourth. $2sw - s^2 = 8$ $2sw - s^2 = d^2 - 10$ From which it follows: $d = \sqrt{18} = 3\sqrt{2}$
14			The area is sum of the angles - π times the radius squared. $\frac{\pi}{3} + \frac{\pi}{2} + \frac{2\pi}{3} - \pi = \frac{3\pi}{2} - \pi = \frac{\pi}{2}$ $A = \frac{\pi}{2}(2)^2 = 2\pi$ sq cm.
	15	15	$12x - 2y^2 + 28y - 2x^2 = 84$ is equivalent to $(x-3)^2 + (y-7)^2 = 16$. The area is $A = \pi r^2 = 16\pi$
15			The curves cross at (1,1) and so the interval must be broken into two segments. $A = \int_0^1 (x - x^2)dx + \int_1^2 (x^2 - x)dx$ $= \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 + \frac{x^3}{3} - \frac{x^2}{2}\Big|_1^2 = \frac{1}{6} + \frac{5}{6} = 1$