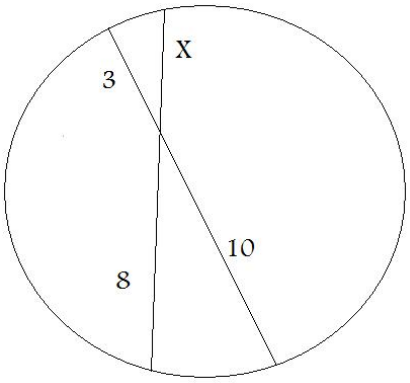
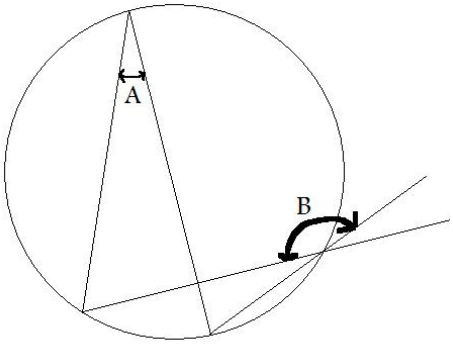
2008 – 2009 Log1 Contest Round 1
Theta Circles

Name: _____

Leave answers in terms of π and fractions in reduced common form.

4 points each		
1	Find the area of a circle with diameter 30.	
2	Find the area of a circle with circumference 12π.	
3	Find the volume of a sphere with maximum chord length 13.	
4	For a circle of area 12π, find the area of sector of arc length 60°.	
5	A triangle is inscribed in a circle by connecting points A, B and C on the circle. If the length of the chord from A to C is equal to twice the circle's radius, find the measure, in degrees, of the angle formed at point B.	

5 points each		
6	Two chords are drawn in a circle, both originating from the same point, A, and ending at the separate points B and C. If the two chords form a 110° arc on the circle from point B to C, what is the measure of angle BAC?	
7	What is the maximum number of points of intersection are there for a set of four distinct circles?	
8	Find the length of the chord segment "x". 	
9	A dart board has radius 9 and a bull's eye of radius 3. A dart thrown at the board has an 80% chance of hitting it at all and will land on a point at random of the board if it does hit. What is the probability of throwing a dart and hitting the board but missing the bull's eye?	
10	An isosceles trapezoid with bases 8 and 18 circumscribes a circle. Find the area of the trapezoid.	

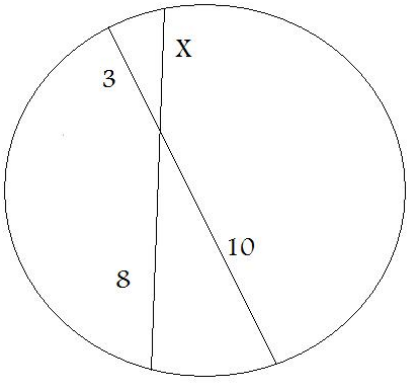
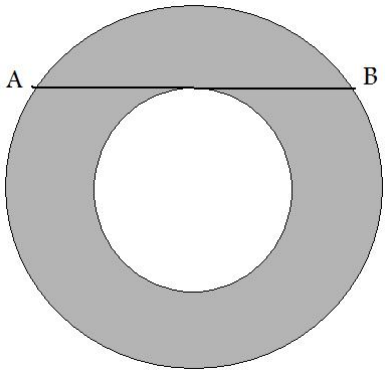
6 points each		
11	Two circles of radius 6 are drawn with their centers $6\sqrt{3}$ units apart. What is the area of the intersection between them?	
12	In the circle below, if the measure of angle A is 30 degrees, then what is the measure of angle B?  	
13	Four points are drawn in a circle such that lines drawn connecting them form a convex quadrilateral. Labeled clockwise, A, B, C and D are the degree measures of the four angles formed in the circle (starting from the top left corner) Evaluate: $(A + C)(B + D)$	
14	What is the surface area of the cylinder (with equal base diameter and height) inscribed in a sphere that is inscribed in a cube with side length 12?	
15	What is the ratio of the area of a square inscribed in a circle to the area of the square circumscribing the same circle (use a:b notation)?	

2008 – 2009 Log1 Contest Round 1
Alpha Circles

Name: _____

Leave answers in terms of π and fractions in reduced common form.

	4 points each	
1	Find the area of a circle with diameter 30.	
2	The area of a sector of a circle with a central angle of 2 radians is $16m^2$. What is the radius of the circle?	
3	Find the volume of a sphere with maximum chord length 13.	
4	For a circle of area 12π, find the area of sector of arc length $60°$.	
5	Two chords are drawn in a circle, both originating from the same point, A, and ending at the separate points B and C. If the two chords form a $110°$ arc on the circle from point B to C, what is the measure of angle BAC?	

	5 points each	
6	What is the maximum number of points of intersection are there for a set of four distinct circles?	
7	Find the length of the chord segment "x". 	
8	In the figure below, the two circles are concentric and the chord of length 32 drawn from A to B is tangent to the inner circle. Find the area of the shaded region. 	
9	A dart board has radius 9 and a bull's eye of radius 3. A dart thrown at the board has an 80% chance of hitting it at all and will land on a point at random of the board if it does hit. What is the probability of throwing a dart and hitting the board but missing the bull's eye?	
10	An isosceles trapezoid with bases 8 and 18 circumscribes a circle. Find the area of the trapezoid.	

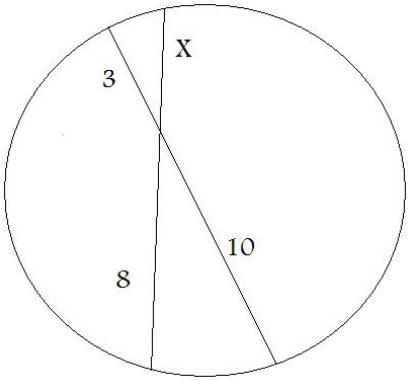
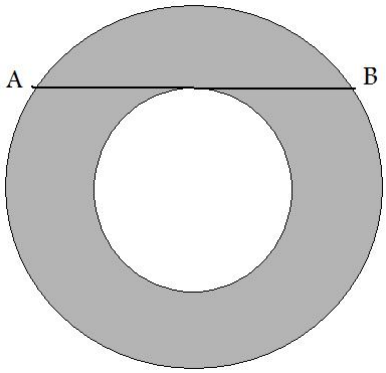
6 points each		
11	Two circles of radius 6 are drawn with their centers $6\sqrt{3}$ units apart. What is the area of the intersection between them?	
12	A rocket flies through a Cartesian coordinate system. Its x-coordinate is given by the equation $x = 2\cos T$ and its y-coordinate is given by the equation $y = 5\sin T$ where T is the number of seconds that the rocket has been flying starting from 0. Find the area of the figure traced by the rocket's motion.	
13	Four points are drawn in a circle such that lines drawn connecting them form a convex quadrilateral. Labeled clockwise, A, B, C and D are the degree measures of the four angles formed in the circle (starting from the top left corner) Evaluate: $(A + C)(B + D)$	
14	What is the surface area of the cylinder (with equal base diameter and height) inscribed in a sphere that is inscribed in a cube with side length 12?	
15	What is the ratio of the area of a square inscribed in a circle to the area of the square circumscribing the same circle (use a:b notation)?	

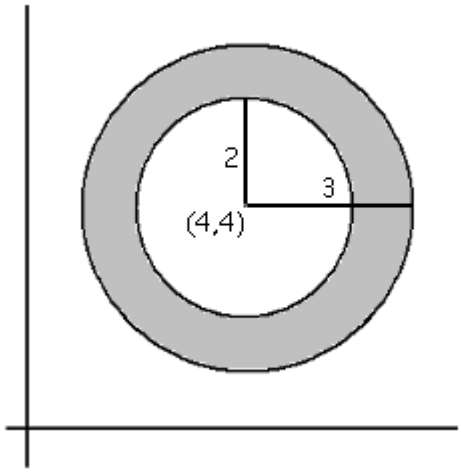
2008 – 2009 Log1 Contest Round 1
Mu Circles

Name: _____

Leave answers in terms of π and fractions in reduced common form.

	4 points each	
1	Find the area of a circle with diameter 30.	
2	The area of a sector of a circle with a central angle of 2 radians is $16m^2$. What is the radius of the circle?	
3	Find the volume of a sphere with maximum chord length 13.	
4	For a circle of area 12π, find the area of sector of arc length $60°$.	
5	A triangle is inscribed in a circle by connecting points A, B and C on the circle. If the length of the chord from A to C is equal to twice the circle's radius, find the measure, in degrees, of the angle formed at point B.	

	5 points each	
6	What is the maximum number of points of intersection are there for a set of four distinct circles?	
7	Find the length of the chord segment "x". 	
8	In the figure below, the two circles are concentric and the chord of length 32 drawn from A to B is tangent to the inner circle. Find the area of the shaded region. 	
9	A dart board has radius 9 and a bull's eye of radius 3. A dart thrown at the board has an 80% chance of hitting it at all and will land on a point at random of the board if it does hit. What is the probability of throwing a dart and hitting the board but missing the bull's eye?	
10	Two circles of radius 6 are drawn with their centers $6\sqrt{3}$ units apart. What is the area of the intersection between them?	

6 points each		
11	Air is being pumped into a spherical balloon at a rate of 100 cubic centimeters per second. Assuming incompressibility of the air, how fast is the radius of the balloon increasing when the diameter is 50cm?	
12	A rocket flies through a Cartesian coordinate system. Its x-coordinate is given by the equation x = 2cosT and its y-coordinate is given by the equation y = 5sinT where T is the number of seconds that the rocket has been flying starting from 0. Find the area of the figure traced by the rocket's motion.	
13	A rocket flies through a Cartesian coordinate system. Its x-coordinate is given by the equation x = 2cosT and its y-coordinate is given by the equation y = 5sinT, where T is the number of seconds that the rocket has been flying starting from 0. What angle does the rocket's path make with the y-axis the rocket when its acceleration in the positive y-direction is greatest?	
14	What is the surface area of the cylinder (with equal base diameter and height) inscribed in a sphere that is inscribed in a cube with side length 12?	
15	 The two circles shown above are concentric and have radii of two and three. What is the volume of the shaded region if the figure above is rotated around the y-axis?	

2008 – 2009 Log1 Contest Round 1
Circles Answers

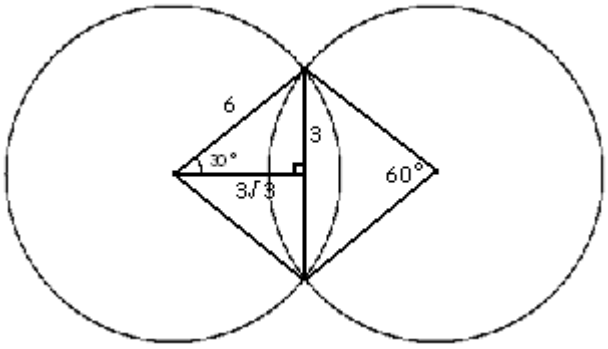
Theta Answers	
1	225π
2	36π
3	$\frac{2197}{6}\pi$
4	2π
5	90°
6	55°
7	12
8	$\frac{15}{4}$
9	$\frac{32}{45}$
10	156 [$units^2$]
11	12π -18√3
12	150°
13	32400
14	108π
15	1:2

Alpha Answers	
1	225π
2	4m
3	$\frac{2197}{6}\pi$
4	2π
5	55°
6	12
7	$\frac{15}{4}$
8	256π
9	$\frac{32}{45}$
10	156 [$units^2$]
11	12π -18√3
12	10π
13	32400
14	108π
15	1:2

Mu Answers	
1	225π
2	4m
3	$\frac{2197}{6}\pi$
4	2π
5	90°
6	12
7	$\frac{15}{4}$
8	256π
9	$\frac{32}{45}$
10	12π -18√3
11	$\frac{1}{25\pi}[m/s]$
12	10π
13	90°
14	108π
15	$40\pi^2$

2008 – 2009 Log1 Contest Round 1
Circles Solutions

Th	Al	Mu	Solution
1	1	1	Since the diameter is 30, then the radius is 15. So by applying 15 to the formula for the radius of a circle: $A = \pi r^2$ $A = 225\pi$
2			The radius is $\frac{12}{2} = 6$, so by squaring it and multiplying it by pi, we get 36π.
	2	2	$A = \pi(r^2)(\frac{\theta}{2\pi})$ $16 = \pi(r^2)(\frac{2}{2\pi})$ $16 = r^2$ $r = 4$
3	3	3	$r = \frac{13}{2}$ $V = \frac{4}{3}\pi(\frac{13}{2})^3 = \frac{2197}{6}\pi$
4	4	4	The area will be (60/360) or 1/6 of the circle, so the answer is 2π, or 1/6 of 12π.
5		5	Since the chord is twice the radius, then it is the diameter, because any triangle inscribed in a semicircle must be a right triangle so the angle measures 90 degrees.
6	5		An inscribed angle measures half its intersected arc so angle BAC is (.5)(110), or 55°.
7	6	6	Each circle can intersect with each other circle a maximum of 2 times. There are 4C2 or 6 ways to choose a pair of circle and each pair can intersect twice, so 12 possible intersections.
8	7	7	The intersection of two chords in a circle gives two segments on each chord. When the measures of the two segments on one chord are multiplied, the result is equal to the product of the measures of the segments of the other chord. 3(10) = 8x or x = 30/8 = 15/4.
	8	8	The area within in an annulus containing a chord tangent to the inner circle is pi times the square of half the chord length. This can be seen by dropping a perpendicular line to the chord from the center. Half the chord squared is then R^2-r^2. $A = \pi(\frac{1}{2}\text{ chord length})^2$
9	9	9	The probability of hitting the board (but not the bull's eye) is: P(hitting board)(Area of non-bull's eye/Area of dartboard) $\frac{4}{5}(\frac{81-9}{81}) = \frac{32}{45}$
10	10		First, the slant height can be found by using the fact that each tangent to a circle from an exterior point is equal and by symmetry, the points of tangency to the bases must bisect the bases. The other two sides are then 13. By using the 5, 12, 13 Pythagorean triple, the height of the trapezoid can be determined to be 12. Averaging the bases and multiplying by 12 gives an area of 156 square units

11	11	10	 The circles form an equilateral triangle with sides length 6 and median length 3√3 units. The 60 degree sector of the circle has area 6 pi and the triangle has area 9√3. This means the area of the intersection is 2(6π -9√3) or 12π -18√3
		11	$r = 25$ $V = \frac{4\pi}{3} r^3$ $\frac{dV}{dt} = 4\pi r^2 (\frac{dr}{dt})$ $\frac{dV}{dt} = \frac{dV}{dr} \bullet \frac{dr}{dt}$ $\frac{dV}{dt} = 100 = 4\pi r^2 (\frac{dr}{dt})$ $100 = 4(25^2)\pi (\frac{dr}{dt})$ $\frac{dr}{dt} = \frac{100}{4(25^2)\pi} = \frac{1}{25\pi}$
12			Since angle A intercepts the same arc as the supplement of angle B, then angle A is equivalent to angle B's supplement. So, because angle A = 30 degrees, then angle B must be 180 – 30 = 150 degrees.
	12	12	The rocket traces out an ellipse with maximum and minimum radii 5 and 2. This can be seen by dividing the first equation by 2 and the second by 5; then square both equations and add. So the area is: $\pi(r_1)(r_2)$ $\pi(5)(2)$ Area of an ellipse = 10π
13	13		In a cyclic quadrilateral opposite angles sum to 180 degrees. So the answer is $180 \bullet 180 = 32400$.
		13	The second derivative of the equation for the y position of the rocket gives: y'' = -sinT. This means acceleration maxes at the point where y displacement is at a minimum. Since the rocket traces an elliptical path and is oriented with its major and minor axes orthogonal to the x and y axis, the velocity will be parallel to the x-axis and form a 90 degree angle with the y-axis.
14	14	14	Radius of sphere = 6 Radius of cylinder = $3\sqrt{2}$ Height of cylinder = $6\sqrt{2}$ Surface Area = $2\pi(3\sqrt{2})^2 + 2\pi(3\sqrt{2})(6\sqrt{2}) = 108\pi$

15	15		The diagonal length of the small square is equal to the side length of the larger square. This puts the squares side lengths at a ratio of 1 to the square root of 2 and their areas in a ratio of 1 to 2.
		15	Volume of a torus (donut): The volume of a torus can be determined from the Theorem of Pappus, which states that the volume of a rotated figure can be obtained by multiplying the radius of rotation from the centroid of the rotated figure by the area of the rotated figure by 2π . In other words: $V = 2\pi RA$ Where : $R =$ radius of rotation = 4 $A =$ area of annulus = 5π The radius of rotation is the distance between the centroid of the annulus and the axis of rotation, which is simply the y-component of the coordinates of the center of the annulus. The area of the annulus can be found by subtracting the area of the larger circle by the area of the smaller circle. So: $V = 2\pi RA$ $= 2\pi(4)(3^2\pi - 2^2\pi)$ $= 2\pi(4)(5\pi)$ $= 40\pi^2$